Formulas

Arithmetic common difference $= d$

recursive: $a_1 = a$ (first)

$a_n = a_{n-1} + d$ (n^{th})

explicit: $a_1 = a$ (first)

$a_n = a_1 + (n-1) \cdot d$ (n^{th})

Arithmetic mean of 2 numbers,

x & y $= \frac{x+y}{2}$

→ term exactly between 2 other terms

Definitely want recursive formula (arithmetic or geometric) when:

1. you don't know a_1, and
2. you do know d (or r), and
3. you want to find a term in the middle, a_n, when you know a term close to a_n, like a_{n-1} or a_{n+1}

Example: arithmetic sequence, find a_{123} when $a_{122} = 361$ and $d = 5$

*recursive

$$a_n = a_{n-1} + d$$

$n = 123$

$$a_{123} = a_{122} + 5$$
$$= 361 + 5$$
$$= 366$$

arithmetic seq. #22 p.624

$-13, -10.5, -8, -5.5, -3$

② $\frac{-8-13}{2} = -10.5$

③ $\frac{-3-8}{2} = -5.5$

$d = -10.5 - (-13)$
$= -10.5 + 13$
$= 13 - 10.5$
$= 2.5$

① $\frac{-3-13}{2} = \frac{-16}{2} = -8$

Geometric sequence (formulas)

common ratio = r

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{a_n}{a_{n-1}}$

recursive: $a_1 = a$ (first)

$a_n = a_{n-1} \cdot r$ (n^{th})

explicit: $a_1 = a$ (first)

$a_n = a_1 \cdot r^{n-1}$ (n^{th})

geometric mean of x & $y = \pm\sqrt{x \cdot y}$ to find term exactly in between two other terms

WHY ± ?

Two different geometric sequences:

#1 5, -10, 20... #2 5, 10, 20...

$r=-2$ $r=2$

#3 5, -10, 20, -40, . . .

$r=-2$

#29 p. 625 geometric

3, $\underline{\pm 6}$, 12, ... missing term(s)

$\pm\sqrt{3 \cdot 12} = \pm\sqrt{36} = \pm 6$

#34 p. 625

geometric 5, 10, 20, 40 ...

10^{th} term, a_{10}

$a_1 = 5$

$r = 2 \quad \left(= \frac{10}{5} = \frac{20}{10} = \frac{40}{20} \cdots\right)$

formula: $a_n = a_1 \cdot r^{n-1}$

$n = 10 \quad a_{10} = 5 \cdot 2^9$

$= 5 \cdot 512 = 2560$

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1 A sequence begins with the terms $a_1 = 5$ and $a_2 = 10$. What would a_3 be if the the sequence is arithmetic? if it is geometric?

$a_1 = 5$

$a_2 = 10$

$a_3 = ?$

arithmetic 15
$(d=5)$

geometric 20
$(r=2)$

2 Consider the sequence: 1, 5, 2, 6, 3, 7, ...

Which of the following describe the sequence?

neither geom. nor arithm.

3 The first term of a geometric sequence is 3. The fifth term of the same sequence is 768. Which of the following could be the terms of the sequence between the first and the fifth?

$a_1 = 3 \quad a_5 = 768$

$$\underset{a_1}{3},\ \underset{a_2}{\boxed{\pm 12}},\ \underset{a_3}{\boxed{\pm 48}},\ \underset{a_4}{\boxed{}},\ \underset{a_5}{768}$$

* Formula: geometric mean of x & y $= \pm\sqrt{x \cdot y}$

$$a_3 = \pm\sqrt{3 \cdot 768} = \pm 48$$

$$a_2 = \pm\sqrt{3 \cdot 48} = \pm 12$$

4 Profits, P, are equal to sales, S, minus expenses, E. If expenses are equal to travel, T, plus materials, M, which system of equations models this situation?

$$P = S - E$$
$$E = T + M$$

5 A sequence is defined using the recursive formula:

$$a_1 = 20$$
$$a_n = a_{n-1} + 6 \quad \text{where } n \geq 2$$

arithmetic

Which of the following sequences would be generated by the recursive formula above?

20, 26, 32, . . .

6 $x(3)$ $\begin{cases} 3x - y + 5 = 0 \\ 2x + 3y - 4 = 0 \end{cases}$

Standard: $Ax + By = C$

What is the solution to this system of equations?

Elimination:

$$9x - 3y + 15 = 0$$
$$2x + 3y - 4 = 0$$
$$11x + 11 = 0$$
$$11x = -11$$
$$x = -1$$

$$3x - y + 5 = 0$$
$$3(-1) - y + 5 = 0$$
$$-3 - y + 5 = 0$$
$$-y + 2 = 0$$
$$y = 2$$

$(-1, 2)$

6 $\begin{cases} 3x - y + 5 = 0 \\ 2x + 3y - 4 = 0 \end{cases}$

Standard: $\underbrace{Ax + By = C}$

What is the solution to this system of equations?

MATRIX: $3x - y = -5$

$2x + 3y = 4$

$\ldots \xleftarrow{\text{rref}} \begin{bmatrix} 3 & -1 & -5 \\ 2 & 3 & 4 \end{bmatrix}$

Topic: convergence and divergence of sequence

convergence: terms get closer and closer to a single output value as n increases

divergence: not convergent terms either increase or decrease without limit

For each of the graphs below, say which of the descriptions A, B, C, or D applies.

A) converging geometric sequence
B) converging arithmetic sequence
C) diverging geometric sequence
D) diverging arithmetic sequence

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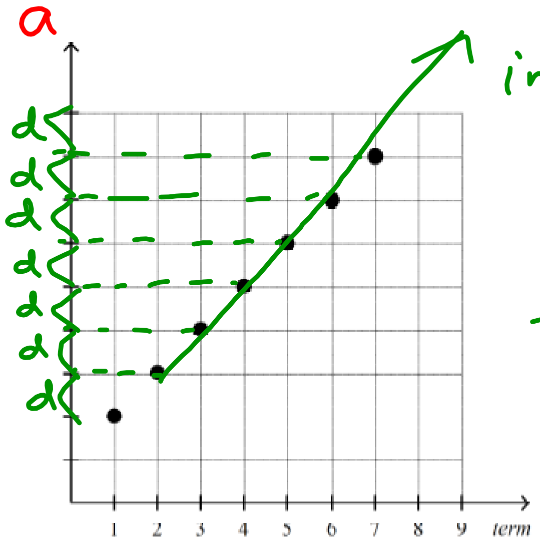

increase without limit
→ not convergent
→ divergent
arithmetic sequences graph on a straight line

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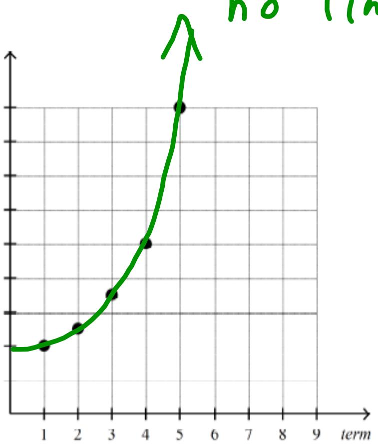

no limit

divergent geometric (not linear)

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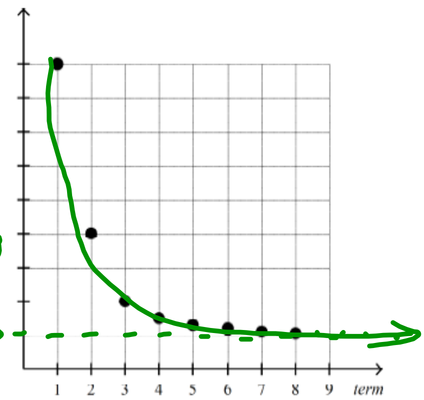

approaching this #

convergent geometric (not linear)

10 Madison had \$20,000 in student loan debt when she graduated from college. The balance increases by 2% each month due to interest. Madison could only make a payment of \$600 per month. Using the formula $b_n = 1.02(b_{n-1}) - 600$ show her balance for the first 5 months after graduation.

\$20,000

interest: 2% each month

payment: \$600 each month

recursive (not arith. or geom.)

$b_n = 1.02(b_{n-1}) - 600$

$n=1$ $b_1 = 20{,}000$

$n=2$ $b_2 = 1.02(20{,}000) - 600 =$ ______

$n=3$ $b_3 = 1.02 b_2 - 600 =$ ______

$\vdots$

11 Terry currently earns \$28,000 per year. Each year Terry receives a \$4,000 increase in salary.

A) Write a recursive formula to model this pattern.

$a_1 = 28{,}000$

$d = 4{,}000$

$a_1 = 28{,}000$

$a_n = a_{n-1} + 4{,}000$

B) Write an explicit formula to model this pattern

$a_1 = 28{,}000$

$a_n = a_1 + (n-1) \cdot 4000$

C) What will Terry's salary be in 12 years?

$n = 13: 28{,}000 + 12 \cdot 4000$

$= 28K + 48K = \$76K$