Ex: solution of a quadratic equation is:

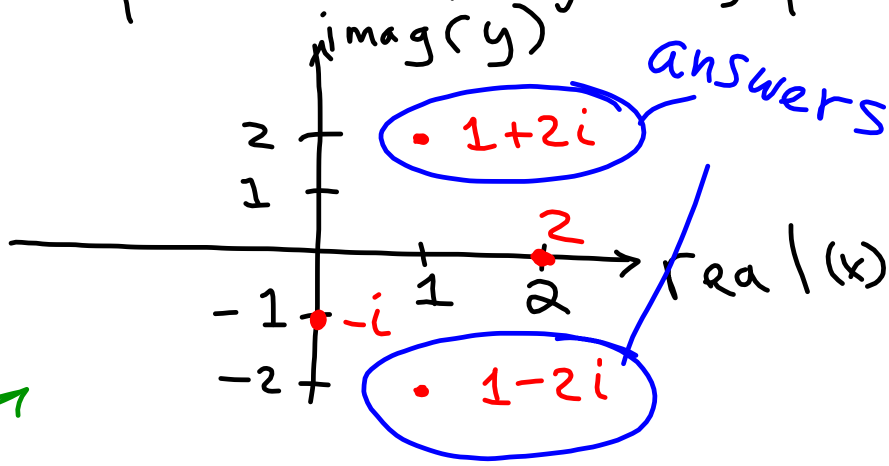
$$x = 1 \pm 2i$$

Q: how do we represent this number in a graph?

$$\underbrace{1}_{\text{real part}} \pm \underbrace{2i}_{\text{imaginary part}}$$

$1 \pm 2i$ is a <u>complex</u> <u>number</u>

real part + imaginary part



complex plane

complex # : $a + bi$

a ↑ real, bi ↑ imag

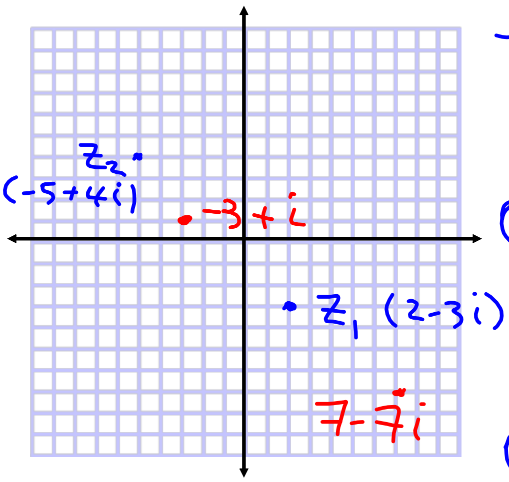
real ↓ x, imag ↓ iy

$x + iy$

$1 + 2i$

1 ↑ real, $2i$ ↑ imag

Topic: adding/subtracting complex #s (also multiplying & dividing)



$z_1 = 2 - 3i$

$z_2 = -5 + 4i$

① $z_1 + z_2$

$(2-3i)+(-5+4i)$

$2-3i-5+4i$

$-3+i$

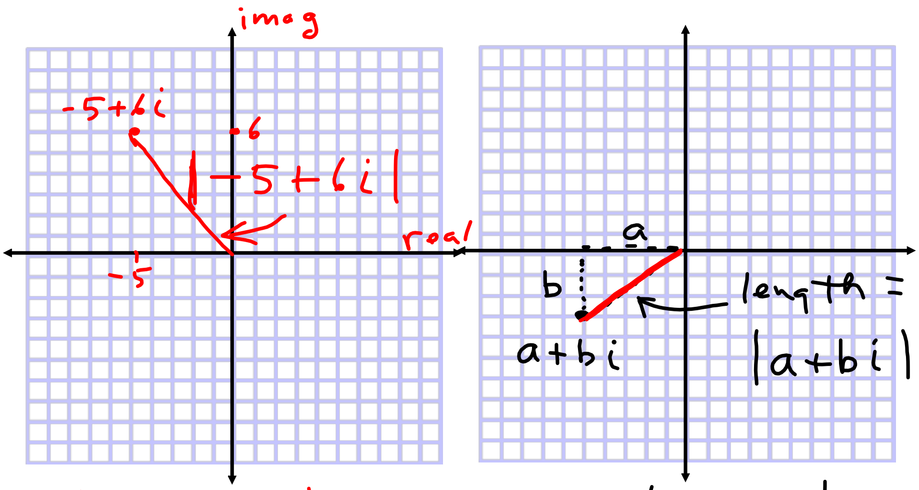
② $z_1 - z_2$

$(2-3i)-(-5+4i)$

$2-3i+5-4i$

$7-7i$

complex planes



$|-5+6i|$
$= \sqrt{25+36} = \sqrt{61}$

absolute value
$= \sqrt{a^2+b^2}$

absolute value
always a real number

multiplying & dividing

if $i = \sqrt{-1}$ then

$$\boxed{i^2 = -1}$$

powers of i

$$i^8 = i^4 = i^0 = 1$$
$$i^5 = i^1 = i$$
$$i^6 = i^2 = -1$$
$$i^7 = i^3 = -i$$

$$i^{537} = i$$

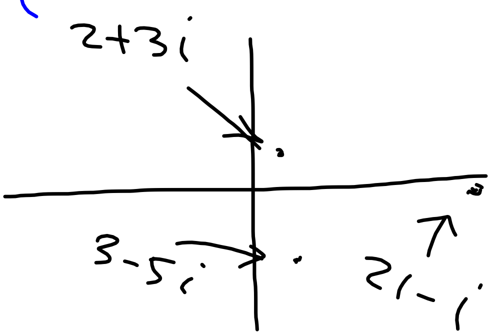
$$4\overline{)537}\quad 134 \text{ r } 1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$$i^4 = i^3 \cdot i = -i \cdot i = -i^2 = -(-1) = 1$$

multiply!

$$(2+3i)(3-5i)$$
$$= 2\cdot 3 + 2(-5i) + 3(3i) + 3i(-5i)$$
$$= 6 - 10i + 9i - 15i^2$$
$$= 6 - 10i + 9i \cancel{-15(-1)}\ +15$$
$$= 21 - i$$



divide!

$$\frac{2+3i}{3-5i} \cdot \frac{3+5i}{3+5i}$$

numerators

$(2+3i)(3+5i) = 6+10i+9i-15 = -9+19i$

denominators

$(3-5i)(3+5i) = 9 \cancel{+15i} \cancel{-15i} -25i^2 = 34$

$+25$

Answer: $\frac{-9+19i}{34}$

$= \frac{-9}{34} + \frac{19}{34} \cdot i$

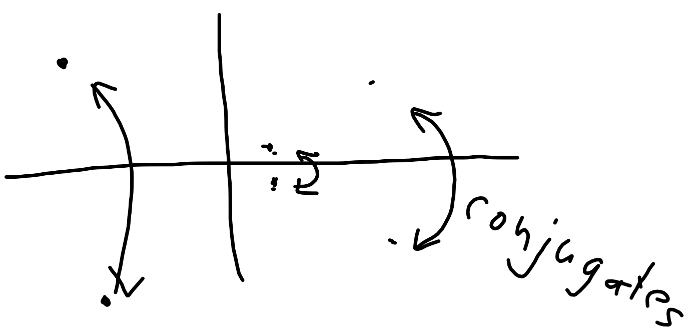
complex conjugates.

- c.c. of z has
  1. same real part
  2. opposite of imag part

in general: c.c. of $a+bi$ is $a-bi$

c.c. $3-5i$ is $3+5i$

c.c. $-1-52i = -1+52i$



property of complex conjugates: when 2 c.c. are multiplied together, the result is a real number.

compare diff. of squares
$a^2 - b^2 = (a+b)(a-b)$

with c.c.
$(a+bi)(a-bi) = a^2 - ab\cdot i + ab\cdot i - b^2 i^2$
$= a^2 + b^2$