In Exercises 1–6, find the average rate of change of the function over each interval.

**1.** $f(x) = x^3 + 1$ (a) 19 (b) 1
(a) $[2, 3]$ (b) $[-1, 1]$

**2.** $f(x) = \sqrt{4x + 1}$ (a) 1 (b) $\frac{7 - \sqrt{41}}{2} \approx 0.298$
(a) $[0, 2]$ (b) $[10, 12]$

**3.** $f(x) = e^x$
(a) $[-2, 0]$ (b) $[1, 3]$

**4.** $f(x) = \ln x$
(a) $[1, 4]$ (b) $[100, 103]$

**5.** $f(x) = \cot t$ (a) $-\frac{4}{\pi} \approx -1.273$ (b) $-\frac{3\sqrt{3}}{\pi} \approx -1.654$
(a) $[\pi/4, 3\pi/4]$ (b) $[\pi/6, \pi/2]$

**6.** $f(x) = 2 + \cos t$ (a) $-\frac{2}{\pi} \approx -0.637$ (b) 0
(a) $[0, \pi]$ (b) $[-\pi, \pi]$

#5 $f(t) = \cot t$

(a) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$$\text{average rate of change} = \frac{f\left(\frac{3\pi}{4}\right) - f\left(\frac{\pi}{4}\right)}{\frac{3\pi}{4} - \frac{\pi}{4}}$$

$$= \frac{\cot\left(\frac{3\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)}{\pi/2}$$

$$= \frac{2\left[\cot\left(\frac{3\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)\right]}{\pi}$$

$\cot\left(\frac{3\pi}{4}\right) = -1 \quad \cot\left(\frac{\pi}{4}\right) = 1$

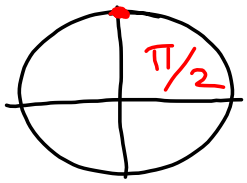
$$= \frac{2(-1-1)}{\pi} = -\frac{4}{\pi}$$

(b) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

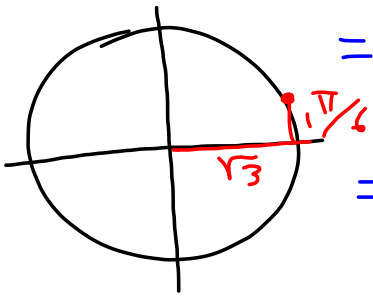
average rate of change $= \dfrac{\cot\left(\frac{\pi}{2}\right) - \cot\left(\frac{\pi}{6}\right)}{\frac{\pi}{2} - \frac{\pi}{6}}$

$\frac{x}{y} = 0$

$= \dfrac{\cot\left(\frac{\pi}{2}\right) - \cot\left(\frac{\pi}{6}\right)}{\pi/3}$




$= \dfrac{3\left(\cot\left(\frac{\pi}{2}\right) - \cot\left(\frac{\pi}{6}\right)\right)}{\pi}$ (0)




$= \dfrac{3(-\sqrt{3})}{\pi}$

$= -\dfrac{3\sqrt{3}}{\pi}$

In Exercises 9–12, at the indicated point find

(a) the slope of the curve, 9: −4 10: −2

(b) an equation of the tangent, and

(c) an equation of the normal.

(d) Then draw a graph of the curve, tangent line, and normal line in the same square viewing window.

**9.** $y = x^2$ at $x = -2$ **10.** $y = x^2 - 4x$ at $x = 1$

(b) tangent @ $x = -2$

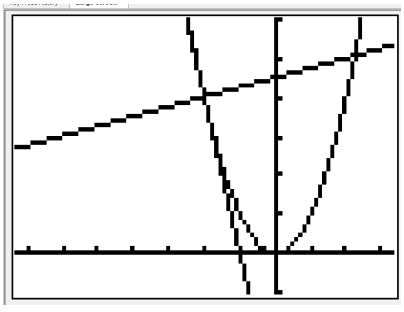
$y = y(-2) + m(x - (-2))$

$y = 4 - 4(x + 2)$

(c) normal @ $x = -2$

$y = y(-2) + \frac{1}{4}(x + 2)$

$= 4 + \frac{1}{4}(x + 2)$



#10 $y = x^2 - 4x$ @ $x = 1$

$m = -2$

$y(1) = -3$

(b) tangent @ $x = 1$

$y = y(1) + m(x-1)$

$= -3 - 2(x-1)$

(c) normal @ $x = 1$

$y = y(1) - \frac{1}{m}(x-1)$

$y = -3 + \frac{1}{2}(x-1)$