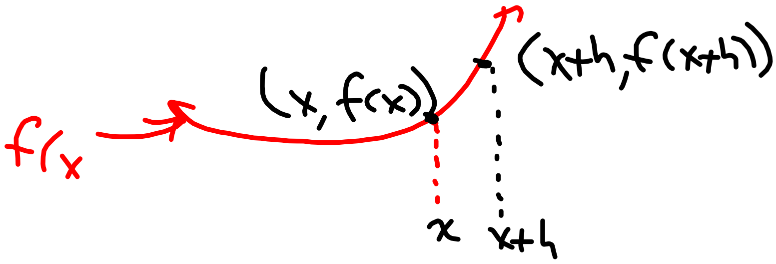
Topic: definition of the derivative as a limit of a difference quotient.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \qquad \frac{\Delta y}{\Delta x}$$



Let $f(x) = 3x^2 - 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 5 - (3x^2 - 5)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2+2hx+h^2) \cancel{-5} - 3x^2 \cancel{+5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} \frac{h(6x+3h)}{h}$$

$$= \lim_{h \to 0} (6x+3h)$$

$$= \lim_{h \to 0} 6x + \lim_{h \to 0} 3h$$

$$= 6x + 0$$

if $f(x) = 3x^2 - 5$ $f'(x) = 6x$

the "slope" (instantaneous rate of change) of

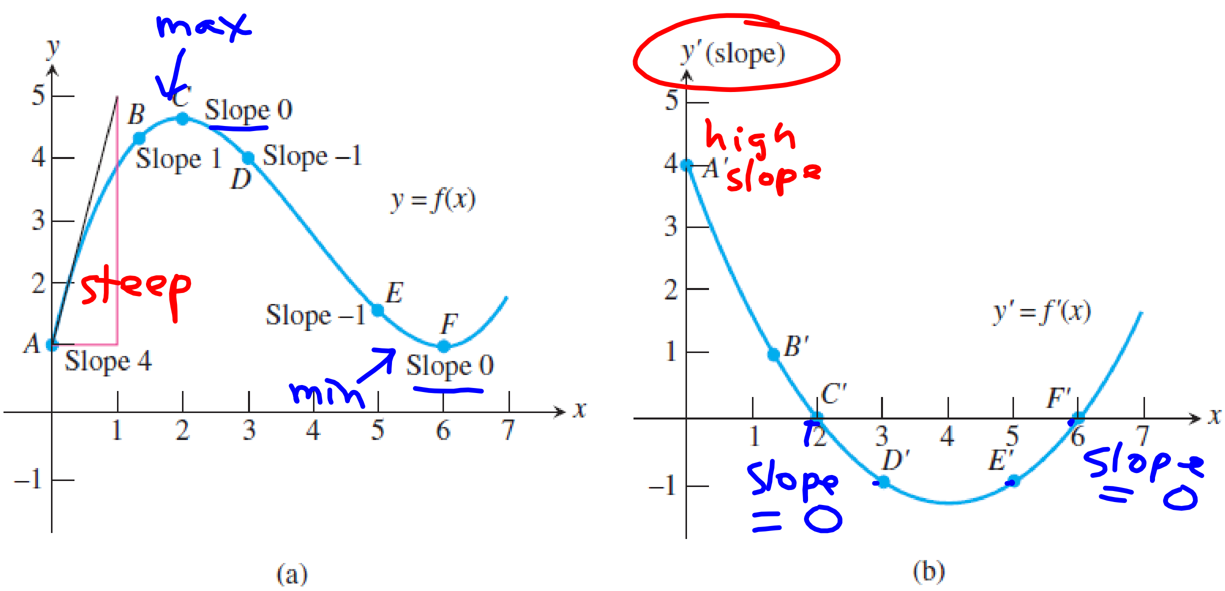
$f(x) = 3x^2 - 5$

is $\underline{6x}$ for every x

if $x = -\frac{1}{2}$ slope $= 6(-\frac{1}{2}) = -3$

if $x = 2$ slope $= 6(2) = 12$

if $x = 0$ slope $= 6(0) = 0$



EXAMPLE 4 Graphing f from f'

Sketch the graph of a function f that has the following properties:

i. $f(0) = 0$;

ii. the graph of f', the derivative of f, is as shown in Figure 3.4;

iii. f is continuous for all x.

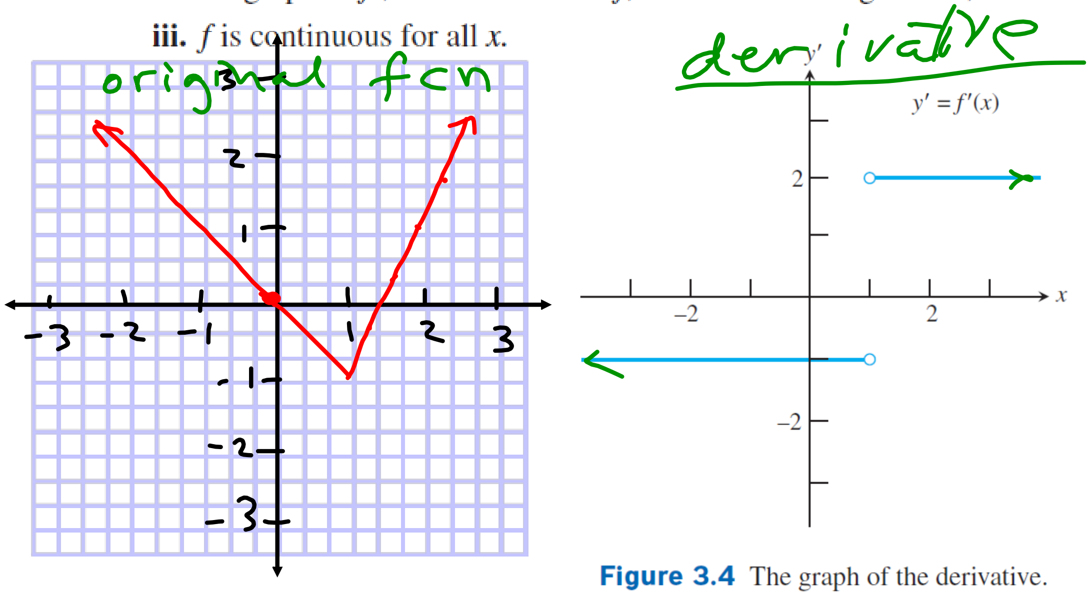


Figure 3.4 The graph of the derivative. (Example 4)