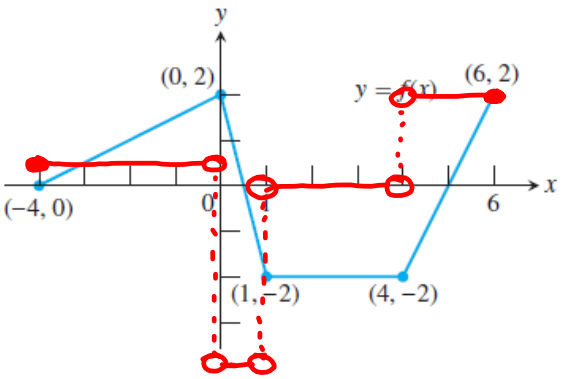
26. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end.

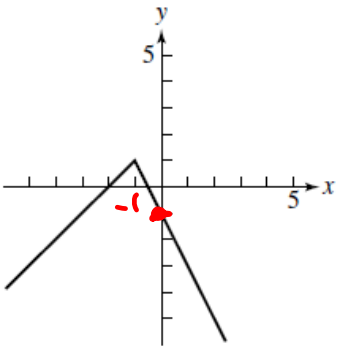


y = f'(x)

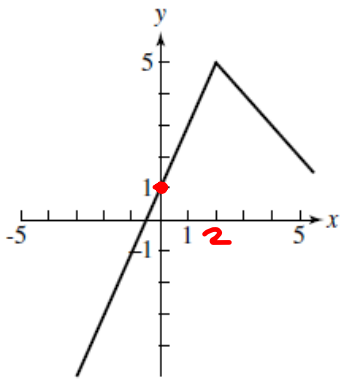
(a) Graph the function's derivative.

(b) At what values of x between $x = -4$ and $x = 6$ is the function not differentiable? $x = 0, 1, 4$

27



28.



In Exercises 1–5, tell whether the limit could be used to define $f'(a)$ (assuming that f is differentiable at a).

1. $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ Yes

2. $\lim_{h\to 0} \dfrac{f(a+h)-f(h)}{h}$ No

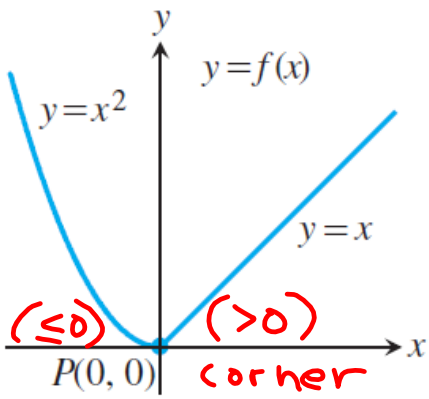
3. $\lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$ Yes

4. $\lim_{x\to a} \dfrac{f(a)-f(x)}{a-x}$ Yes

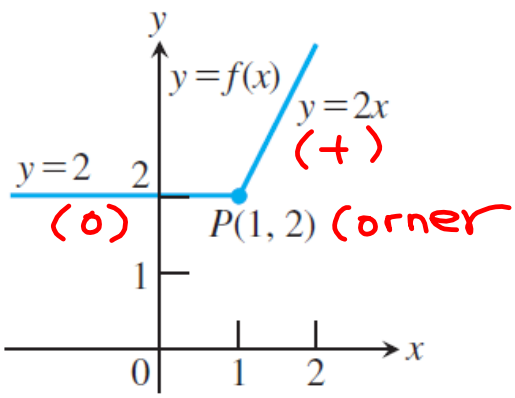
5. $\lim_{h\to 0} \dfrac{f(a+h)-f(a-h)}{h}$ No

not a difference

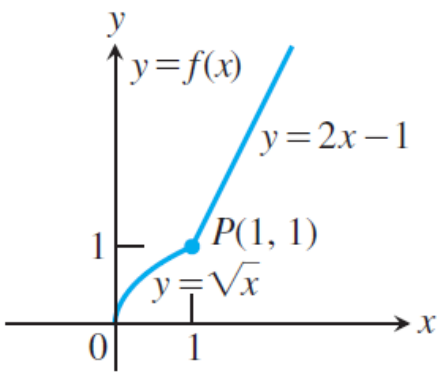
1.



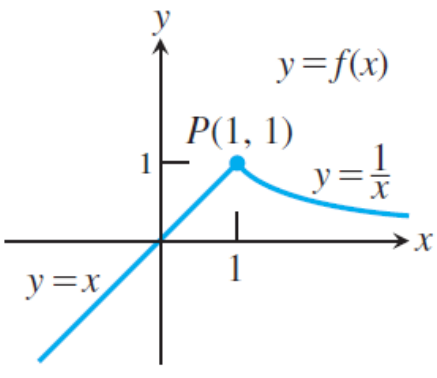
2.



3.



4.



Rules — and notation

$\frac{dy}{dx}$ "derivative of y with respect to x"

$f'(x)$ " first derivative of $f(x)$" (with respect to x is implied

y' "y prime" shorthand for $\frac{dy}{dx}$ or $\frac{dy}{dt}$

$\frac{d}{dx}(\text{something})$ "d-dx of something" (e.g. expression)

Constant fcn

derivative of a constant = 0

k constant, $\frac{d}{dx}(k) = 0$

(b/c derivative is a rate of change)

power rule (all real n)

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

$$\frac{d}{dx}(x^3) = 3 \cdot x^2$$

constant multiple rule

k constant, $\frac{d}{dx}(k \cdot f(x)) = k \cdot f'(x)$

sum and difference rule(s)

$$\frac{d}{dx}\left(f(x)+g(x)\right) = f'(x)+g'(x)$$

"differentiating" means "taking the derivative of"

if $f(x)$ has a horizontal tangent, $f'(x)=0$

find a horiz. tangent by setting $f'(x)=0$

Example 1:

$$p = t^3 + 6t^2 - \frac{5}{3}t + 16$$

sum

$$p'(t) = 3t^2 + 6\cdot 2t - \frac{5}{3}\cdot 1 + 0$$

power power power constant

const. multiple. const. multiple.

$$= 3t^2 + 12t - \frac{5}{3}$$

Example 2:

$y = x^4 - 2x^2 + 2$

horizontal tangents?

$y' = 4x^3 - 2 \cdot 2x + 0$

$= 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x+1)(x-1)$

h.t. @ $x = 0, x = \pm 1$