p. 123

In Exercises 1–6, write the expression as a sum of powers of x.

1. $(x^2 - 2)(x^{-1} + 1)$
$x + x^2 - 2x^{-1} - 2$

2. $\left(\dfrac{x}{x^2 + 1}\right)^{-1}$ $x + x^{-1}$

3. $3x^2 - \dfrac{2}{x} + \dfrac{5}{x^2}$
$3x^2 - 2x^{-1} + 5x^{-2}$

4. $\dfrac{3x^4 - 2x^3 + 4}{2x^2}$ $\dfrac{3}{2}x^2 - x + 2x^{-2}$

5. $(x^{-1} + 2)(x^{-2} + 1)$
$x^{-3} + x^{-1} + 2x^{-2} + 2$

6. $\dfrac{x^{-1} + x^{-2}}{x^{-3}}$ $x^2 + x$

#2: $\dfrac{x^2+1}{x} = \dfrac{x^2}{x} + \dfrac{1}{x}$
$= x + x^{-1}$

p. 124

1. $y = -x^2 + 3$ $dy/dx = -2x$

2. $y = \dfrac{x^3}{3} - x$ $dy/dx = x^2 - 1$

3. $y = 2x + 1$ $dy/dx = 2$

4. $y = x^2 + x + 1$ $dy/dx = 2x + 1$

5. $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} + x$
$dy/dx = x^2 + x + 1$

6. $y = 1 - x + x^2 - x^3$
$dy/dx = -1 + 2x - 3x^2$

In Exercises 7–12, find the horizontal tangents of the curve.

7. $y = x^3 - 2x^2 + x + 1$ At $x = 1/3, 1$

8. $y = x^3 - 4x^2 + x + 2$

9. $y = x^4 - 4x^2 + 1$ At $x = 0, \pm\sqrt{2}$

10. $y = 4x^3 - 6x^2 - 1$ At $x = 0, 1$

11. $y = 5x^3 - 3x^5$ At $x = -1, 0, 1$

12. $y = x^4 - 7x^3 + 2x^2 + 15$

#8, 12: Quadratic Formula

#9 $(x^2-2) = 0 \Rightarrow x = \pm\sqrt{2}$

to handle $\frac{x^3}{3}$ as a constant-multiple rule:

$\frac{1}{3}x^3$

#8 p. 124 horiz. tangent

$y = x^3 - 4x^2 + x + 2$

$y' = 3x^2 - 4 \cdot 2x + 1 + 0$

$y' = 3x^2 - 8x + 1$

(the derivative)

for horizontal tangent, $y' = 0$

$3x^2 - 8x + 1 = 0$

Q.F. $x = \frac{8 \pm \sqrt{64-12}}{6}$

$= \frac{8 \pm 2\sqrt{13}}{6}$

$= \frac{4 \pm \sqrt{13}}{3}$

dissect $\frac{d}{dx}(4x^2)$

$$\frac{d}{dx}(4x^2) = 4 \cdot \frac{d}{dx}(x^2)$$

b/c constant multiple

$$= 4 \cdot 2x^1$$

b/c power rule

$$= 8x$$

Topic: product, quotient rule.

have: polynomials

today we will have:

products: $f(x) = u(x) \cdot v(x)$

quotients : $f(x) = \frac{u(x)}{v(x)}$

where $v(x) \neq 0$

$\pi \cdot x^2$ product? yes, but use const. mult.

$\frac{x^2}{\pi}$ quotient? yes, but use const. mult.

$$= \frac{1}{\pi} \cdot x^2$$

sidebar: constant multiple rule

in general, if $f(x)$ is differentiable, then

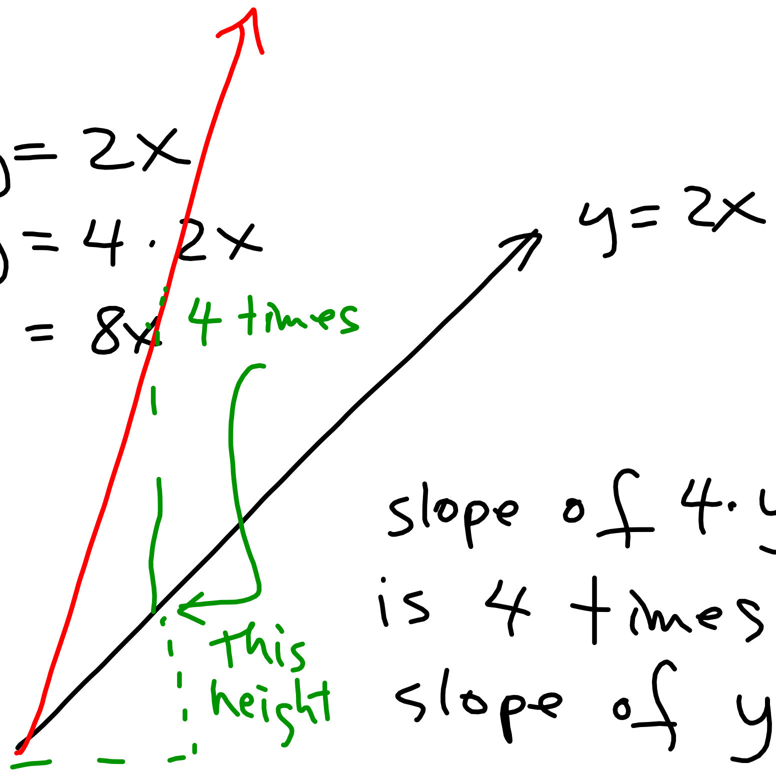
$$\frac{d}{dx}\left(k\cdot f(x)\right) = k\cdot \underbrace{\frac{d}{dx}\left(f(x)\right)}_{f'(x)}$$

derivative of

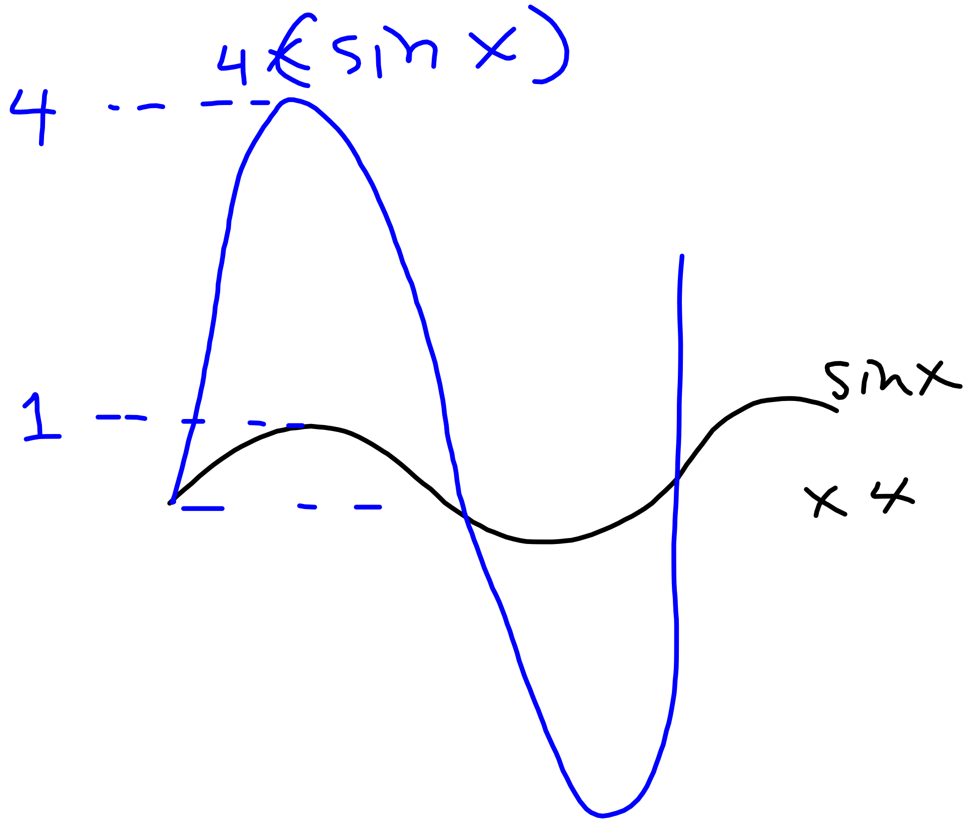
f

k constant

$$y = 2x$$
$$4\cdot y = 4\cdot 2x$$
$$= 8x$$



slope of $4\cdot y$ is 4 times slope of y



product rule.

$$\frac{d}{dx}\left(u(x)\cdot v(x)\right) = \frac{du}{dx}\cdot v(x) + u(x)\cdot\frac{dv}{dx}$$

short version: $\frac{d}{dx}(u\cdot v) = u'v + uv'$

quotient rule.

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{\frac{du}{dx}\cdot v(x) - u(x)\cdot\frac{dv}{dx}}{(v(x))^2}$$

short version: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$u(x) = x^2 - 3 \qquad v(x) = x^2+1$

$u' = 2x \qquad v' = 2x$

let $f(x) = u(x)\cdot v(x) = (x^2-3)(x^2+1)$

$f'(x) = u'v + uv'$

$= 2x(x^2+1) + (x^2-3)2x$

$= 2x^3 + 2x + 2x^3 - 6x$

...

let $g(x) = \dfrac{u(x)}{v(x)} = \dfrac{x^2-3}{x^2+1}$

$g'(x) = \dfrac{u'v - uv'}{v^2}$

$= \dfrac{2x(x^2+1) - (x^2-3)2x}{(x^2+1)^2}$

power rule:

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}(x^2) = 2x^1 = 2x$$

$$\frac{d}{dx}(x^{37}) = 37x^{36}$$