14. Let $y = (x^2 + 3)/x$. Find dy/dx (**a**) by using the Quotient Rule, and (**b**) by first dividing the terms in the numerator by the denominator and then differentiating.

(a) $y = \dfrac{x^2+3}{x}$ $\quad u = x^2+3 \quad v = x$
$\quad u' = 2x \quad v' = 1$

$$y' = \frac{u'v - uv'}{v^2} = \frac{2x(x) - (x^2+3)}{x^2}$$

$$= \frac{2x^2 - x^2 - 3}{x^2} = \frac{x^2-3}{x^2} = 1 - \frac{3}{x^2}$$

(b) $y = \dfrac{x^2+3}{x} = \dfrac{x^2}{x} + \dfrac{3}{x} = x + 3x^{-1}$

$$y' = 1 + 3(-1x^{-2})$$

$$= 1 - \frac{3}{x^2}$$

In Exercises 15–22, find dy/dx. Support your answer graphically.

15. $(x^3 + x + 1)(x^4 + x^2 + 1)$

16. $(x^2 + 1)(x^3 + 1)$ $\quad 5x^4 + 3x^2 + 2x$

17. $y = \dfrac{2x+5}{3x-2}$ $\quad -\dfrac{19}{(3x-2)^2}$

18. $y = \dfrac{x^2 + 5x - 1}{x^2}$ $\quad -\dfrac{5}{x^2} + \dfrac{2}{x^3}$

19. $y = \dfrac{(x-1)(x^2+x+1)}{x^3}$ $\quad \dfrac{3}{x^4}$

20. $y = (1-x)(1+x^2)^{-1}$ $\quad \dfrac{x^2 - 2x - 1}{(1+x^2)^2}$

21. $y = \dfrac{x^2}{1-x^3}$ $\quad \dfrac{x^4 + 2x}{(1-x^3)^2}$

22. $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$ $\quad \dfrac{12 - 6x^2}{(x^2 - 3x + 2)^2}$

#20 $\quad y = \dfrac{1-x}{1+x^2}$ $\quad u = 1-x \quad v = 1+x^2$
$\quad u' = -1 \quad v' = 2x$

$$y' = \frac{u'v - uv'}{v^2} = \frac{-1 - x^2 - 2x(1-x)}{(1+x^2)^2}$$

$$= \frac{-1 - x^2 - 2x + 2x^2}{(1+x^2)^2}$$

$$= \frac{x^2 - 2x - 1}{(1+x^2)^2}$$

24. Suppose u and v are functions of x that are differentiable at $x = 2$ and that $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$. Find the values of the following derivatives at $x = 2$.

(a) $\frac{d}{dx}(uv)$ 2 **(b)** $\frac{d}{dx}\left(\frac{u}{v}\right)$ -10

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ $\frac{10}{9}$ **(d)** $\frac{d}{dx}(3u - 2v + 2uv)$ -12

(a) $\frac{d}{dx}(uv) = u'v + uv' = (-4)(1) + (3)(2) = -4 + 6 = 2$

(b) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2} = \ldots$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{v \cdot u' - uv'}{u^2} = \ldots$

(d) $\frac{d}{dx}(3u - 2v + 2uv) =$

$3u' - 2v' + 2(u'v + uv') =$

$\ldots$

25. Which of the following numbers is the slope of the line tangent to the curve $y = x^2 + 5x$ at $x = 3$? (iii)

i. 24 **ii.** $-5/2$ **iii.** 11 **iv.** 8

26. Which of the following numbers is the slope of the line $3x - 2y + 12 = 0$? (iii)

i. 6 **ii.** 3 **iii.** $3/2$ **iv.** $2/3$

$\frac{3x + 12}{2} = \frac{2y}{2}$

$y = x^2 + 5x$

$y' = 2x + 5$

$6 + 5 = 11$

$\frac{3}{2}x + 6 = y$

In Exercises 27 and 28, find an equation for the line tangent to the curve at the given point.

27. $y = \dfrac{x^3+1}{2x}$, $x = 1$ $\quad y = \frac{1}{2}x + \frac{1}{2}$

28. $y = \dfrac{x^4+2}{x^2}$, $x = -1$ $\quad y = 2x + 5$

point-slope form

$y = 1 + \frac{1}{2}(x-1)$

$= 1 + \frac{1}{2}x - \frac{1}{2} = \frac{1}{2}x + \frac{1}{2}$

$y'\big|_{x=1}$ $\quad y\big|_{x=1}$

$m = \frac{1}{2}$ $\quad y_1 = 1$ $\quad x_1 = 1$

$u = x^3 + 1 \quad v = 2x$

$u' = 3x^2 \quad v' = 2$

$y' = \dfrac{u'v - uv'}{v^2}$

$= \dfrac{3x^2(2x) - (x^3+1)2}{(2x)^2}\Bigg|_{x=1} = \dfrac{6-4}{4} = \dfrac{1}{2}$

#28 $y = \dfrac{x^4+2}{x^2}$ @ $x=-1$

$y = \dfrac{x^4}{x^2} + \dfrac{2}{x^2} = x^2 + 2x^{-2}$

$y' = 2x + 2\cdot(-2x^{-3})$

$= 2x - \dfrac{4}{x^3}$

$y'\big|_{x=-1} = 2(-1) - \dfrac{4}{-1} = -2+4 = 2$

$m = 2$

$y\big|_{x=-1} = \dfrac{(-1)^4+2}{(-1)^2} = \dfrac{3}{1} = 3$

$y_1 = 3$

tangent line:

$y = 3 + 2(x+1)$

$= 3 + 2x + 2 = 2x + 5$

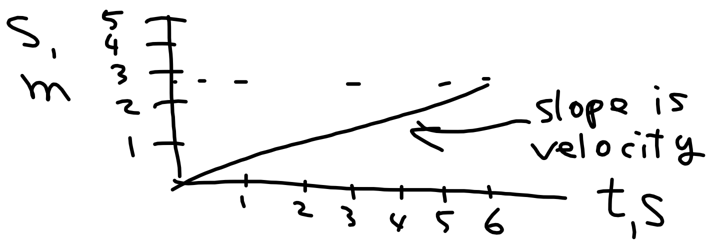
$y = 2x + 5$

topic: velocity, acceleration, speed

derivative = instantaneous rate of change

position: $s(t)$, meters, second's

move at constant rate



velocity is rate of change of position with respect to time

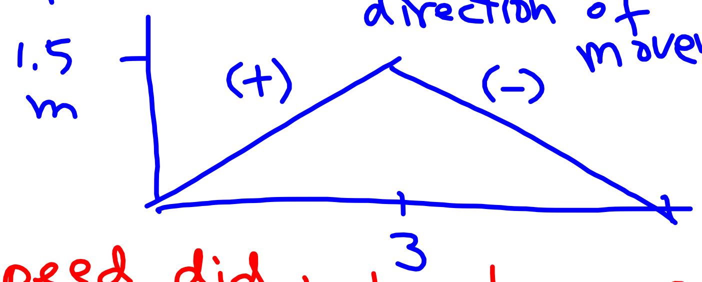
velocity is the derivative of position

$v(t) = s'(t)$

Speed is the absolute value of velocity

Example

velocity implies a direction of movement



Speed did not change

acceleration is the derivative of velocity.

$$a(t) = v'(t) : s''(t)$$

first derivative

2nd derivative, derivative of derivative