2. **(a)** Write the area A of a circle as a function of the circumference C. $A = \frac{C^2}{4\pi}$

(b) Find the (instantaneous) rate of change of the area A with respect to the circumference C. $\frac{dA}{dC} = \frac{C}{2\pi}$

(c) Evaluate the rate of change of A at $C = \pi$ and $C = 6\pi$. 1/2, 3

(d) If C is measured in inches and A is measured in square inches, what units would be appropriate for dA/dC? in^2 /in.

(b) $\frac{dA}{dC} = \frac{d}{dC}\left(\frac{1}{4\pi}C^2\right)$

$= \frac{1}{4\pi} \cdot 2C$

$= \frac{1}{2\pi} C$

(a) $A = \pi r^2$ $\qquad C = 2\pi r$

$= \pi\left(\frac{1}{2\pi}C\right)^2$ $\qquad r = \frac{C}{2\pi}$

$= \frac{\pi}{4\pi^2}C^2 = \frac{1}{4\pi}C^2$

(c) $\left.\frac{dA}{dC}\right|_{C=\pi} = \frac{1}{2\pi} \cdot \pi = \frac{1}{2}$

$\left.\frac{dA}{dC}\right|_{C=6\pi} = \frac{1}{2\pi} \cdot 6\pi = 3$

(d) units $\frac{dA}{dC} = \frac{A \text{ units}}{C \text{ units}} = \frac{\text{in}^2}{\text{in}}$

4. A square of side length s is inscribed in a circle of radius r.

(a) Write the area A of the square as a function of the radius r of the circle. $A = 2r^2$

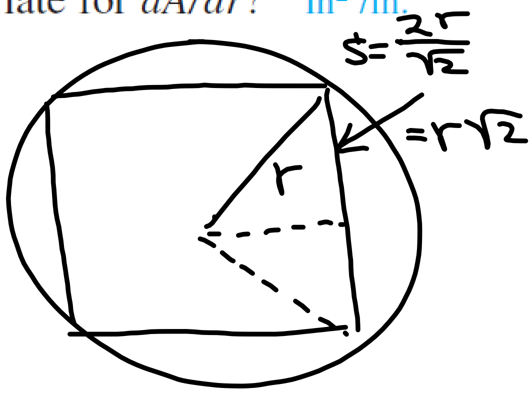
(b) Find the (instantaneous) rate of change of the area A with respect to the radius r of the circle. $\frac{dA}{dr} = 4r$

(c) Evaluate the rate of change of A at $r = 1$ and $r = 8$. 4, 32

(d) If r is measured in inches and A is measured in square inches, what units would be appropriate for dA/dr? in^2 /in.

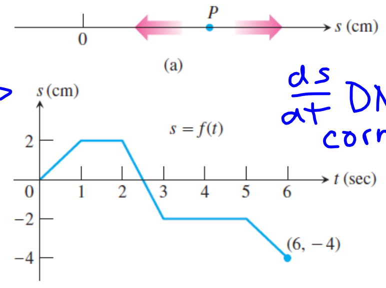
(a) $A = s^2 = (r\sqrt{2})^2$

$= 2r^2$



10. *Particle Motion* A particle P moves on the number line shown in part (a) of the accompanying figure. Part (b) shows the position of P as a function of time t.

position



(b)

$\frac{ds}{dt}$ DNE @ corners: $t = 1, 2, 3, 5$

(a) When is P moving to the left? moving to the right? standing still? See page 140.

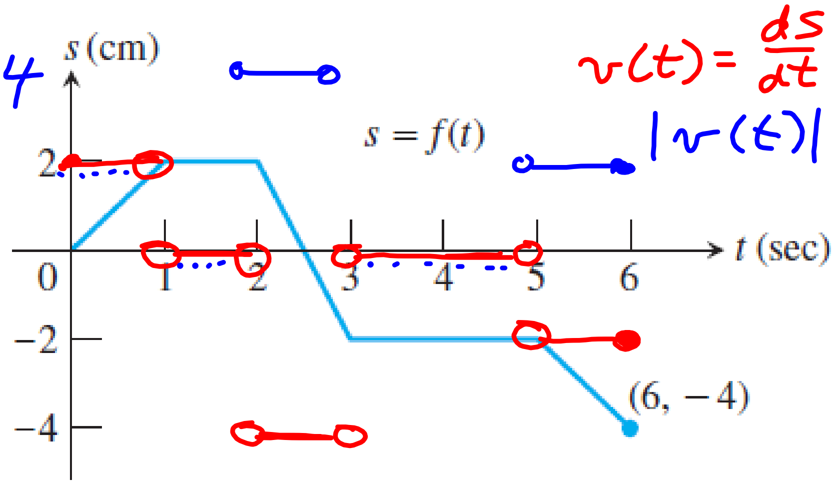
(a) $(2,3) \iff 2 < t < 3$

$(5,6] \iff 5 < t \le 6$

(b) $[0,1) \iff 0 \le t < 1$

(c) $(1,2)$

$(3,5)$



(b)

4

$v(t) = \frac{ds}{dt}$

$|v(t)|$

(b) Graph the particle's velocity and speed (where defined).

13. *Lunar Projectile Motion* A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s = 24t - 0.8t^2$ meters in t seconds.

(a) Find the rock's velocity and acceleration as functions of time. (The acceleration in this case is the acceleration of gravity on the moon.) vel(t) = $24 - 1.6t$ m/sec, accel(t) = -1.6 m/sec^2

(b) How long did it take the rock to reach its highest point? 15 seconds

(c) How high did the rock go? 180 meters

(d) When did the rock reach half its maximum height? About 4.393 seconds

(e) How long was the rock aloft? 30 seconds

$v(0) = 24$ m/s

$s(t) = 24t - 0.8t^2$ m

(a) $v(t) = 24 - 1.6t$ m/s

$a(t) = \frac{dv}{dt} = -1.6$ $\frac{m/s}{s}$

(b) $v(t) = 0$

$24 - 1.6t = 0$

$1.6t = 24$

$t = 15$ s

(c) $s(15) = 24(15) - 0.8(225)$

$= \approx 180$ m

(d) $90 = 24t - 0.8t^2$

(solve)

(e) 30 s

Topic: derivatives of trig fcns.

$\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$ $\frac{d}{dx}(\tan x) = \sec^2 x$

$(= (\sec x)^2)$

$\left(= \left(\frac{1}{\cos x}\right)^2\right)$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

Example: $\frac{d}{dx}(x^2 \cdot \sin x)$

product: $u = x^2 \quad v = \sin x$

$u' = 2x \quad v' = \cos x$

$$\frac{d}{dx}(x^2 \cdot \sin x) = u' \cdot v + u \cdot v'$$

$$= 2x \sin x + x^2 \cos x$$