Topic: Chain Rule for derivatives.

Example 1: $\frac{d}{dx}(\sin x) = \cos x$

Example 2: $\frac{d}{dx}(x^2) = 2x$

Example 3: $\frac{d}{dx}(\sin^2 x) = ?$

composition

$f(g(x))$ f is "outside" fcn $f(x) = x^2$

g is "inside" fcn $g(x) = \sin x$

Rule which allows us to take the derivative of a composition is THE CHAIN RULE.

$$\frac{d}{dx} f(g(x)) = \underbrace{f'(g(x))}_{\text{derivative of } f \text{ with respect to its input, with input held constant}} \cdot g'(x)$$

Example3 $y = \sin^2 x$

$\sin^2 x = f(g(x))$ where

$f(x) =$ square fcn.

$g(x) =$ sine fcn

$$y = (\sin x)^2$$

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

$$= 2 \cdot g(x) \cdot \cos x$$

$$\frac{dy}{dx} = 2 \sin x \cos x$$

Example 4: $\sin(x^2) = y$

$f(g(x))$ $f =$ sine fcn

$g =$ square

$$\frac{dy}{dx} = \underbrace{\cos(x^2)}_{f'(g(x))} \cdot \underbrace{(2x)}_{g'(x)}$$

$$\frac{dy}{dx} = 2x \cos(x^2)$$

Example 5: $\tan(3x^2+1)=y$

$$\frac{dy}{dx}=\sec^2(3x^2+1)\cdot 6x$$

$$\frac{dy}{dx}=6x\sec^2(3x^2+1)$$

Example 6: $y=\sin^2(3x^2+2x)$

$$y=\left[\sin(3x^2+2x)\right]^2$$

$$\frac{dy}{dx}=2\sin(3x^2+2x)\cdot\cos(3x^2+2x)(6x+2)$$

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$f(x) = \sin x$

$g(x) = x^2 + 1$

$h(x) = 7x$

these x's are "place-holders" signifying input

① $f(g(x)) = \sin(x^2+1)$

② $f(g(h(x))) = \sin((7x)^2+1)$

$= \sin(49x^2+1)$

$f(x) = \cos x$

$g(x) = \sqrt{x+2}$

$h(x) = 3x^2$

6. $\sqrt{\cos x + 2} = g(f(x))$

7. $\sqrt{3\cos^2 x + 2} = g(h(f(x)))$

8.

Ex #1 $y = \sin(3x+1)$

$u = 3x + 1$

$y = \sin u$

$\frac{dy}{dx} = \cos u \cdot \frac{du}{dx}$

$= \cos(3x+1) \cdot 3$

#3 $y = \cos(\sqrt{3} \cdot x)$ $u = \sqrt{3} \cdot x$

$\frac{du}{dx} = \sqrt{3}$

$y = \cos u$

$\frac{dy}{dx} = -\sin u \cdot \frac{du}{dx}$

$\frac{dy}{dx} = -\sin(\sqrt{3} \cdot x) \cdot \sqrt{3}$

$\frac{dy}{dx} = -\sqrt{3} \cdot \sin(\sqrt{3} \cdot x)$