Composition: $y = f(g(x))$

f is "outside fcn"

g is "inside fcn"

Chain Rule: $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

also: $\frac{dy}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Examples:

① $y = \tan^2 x$ $\quad f(x) = x^2$, $g(x) = \tan x$

$= (\tan x)^2$

$y' = 2 \cdot \tan x \cdot \sec^2 x$

Chain Rule $\longrightarrow$ $f'(g(x)) \cdot g'(x)$

② $y = \tan(\sin x)$ $\quad f(x) = \tan x$, $g(x) = \sin x$

$y' = \sec^2(\sin x) \cdot \cos x$

Q. Rev. p. 153 # 8

$3\cos x + 6$

$3(\cos x + 2)$

$\cancel{3 \quad g^2(f(x))}$

$h(g(f(x)))$

p. 153 #5 (exercise)

$$y = \left(\frac{\sin x}{1+\cos x}\right)^2$$

$$u = \frac{\sin x}{1+\cos x}$$

$$y = u^2$$

Chain rule $\frac{dy}{dx} = 2u \cdot \frac{du}{dx}$

$u = \sin x \quad v = 1+\cos x$

$u' = \cos x \quad v' = -\sin x$

$$\frac{du}{dx} = \frac{u'v - uv'}{v^2}$$

$$= \frac{(\cos x)(1+\cos x) + \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{(\cos x)(1+\cos x) + \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{1+\cos x}{(1+\cos x)^2}$$

$$\frac{du}{dx} = \frac{1}{1+\cos x}$$

$$\frac{dy}{dx} = 2u \cdot \frac{du}{dx}$$

$$= 2 \cdot \frac{\sin x}{1+\cos x} \cdot \frac{1}{1+\cos x}$$

$$\frac{dy}{dx} = \frac{2\sin x}{(1+\cos x)^2}$$

#6 $y = 5\cot\frac{2}{x}$, $u = \frac{2}{x}$

$= 2x^{-1}$

$y = 5\cot u$

$\frac{du}{dx} = -2x^{-2}$

$= -\frac{2}{x^2}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= -5\csc^2 u \cdot \left(\frac{-2}{x^2}\right)$$

$$= \frac{10\csc^2\left(\frac{2}{x}\right)}{x^2}, \quad x \neq 0$$