# Tangent & Normal Line

need: point $(x_1, y_1)$

slope $\left.\frac{dy}{dx}\right|_{x=x_1}$

or $\frac{-1}{\left.\frac{dy}{dx}\right|_{x=x_1}}$

#45 $y = \sqrt{x^2-2x}$ $x_1 = 3$, $y_1 = \sqrt{3}$

Power Rule

$\frac{d}{dx}(x^n) = n x^{n-1}$

$= (x^2-2x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x^2-2x)^{-1/2} \cdot (2x-2)$

$= \frac{2(x-1)}{2\sqrt{x^2-2x}}$

$\left.\frac{dy}{dx}\right|_{x=3} = \frac{2}{\sqrt{3}}$ = slope of tangent

tangent line:

$y = y_1 + m(x - x_1)$

$y = \sqrt{3} + \frac{2}{\sqrt{3}}(x-3)$

normal line:

$y = \sqrt{3} - \frac{\sqrt{3}}{2}(x-3)$

#46 $y = 4 + \cot x - 2\csc x$

$x = \pi/2$ $\quad x_1 = \frac{\pi}{2}$

$y_1 = 4 + \cot\left(\frac{\pi}{2}\right) - 2\csc\left(\frac{\pi}{2}\right)$

$= 4 + 0 - 2 \cdot 1$

$= 4 - 2 = 2$

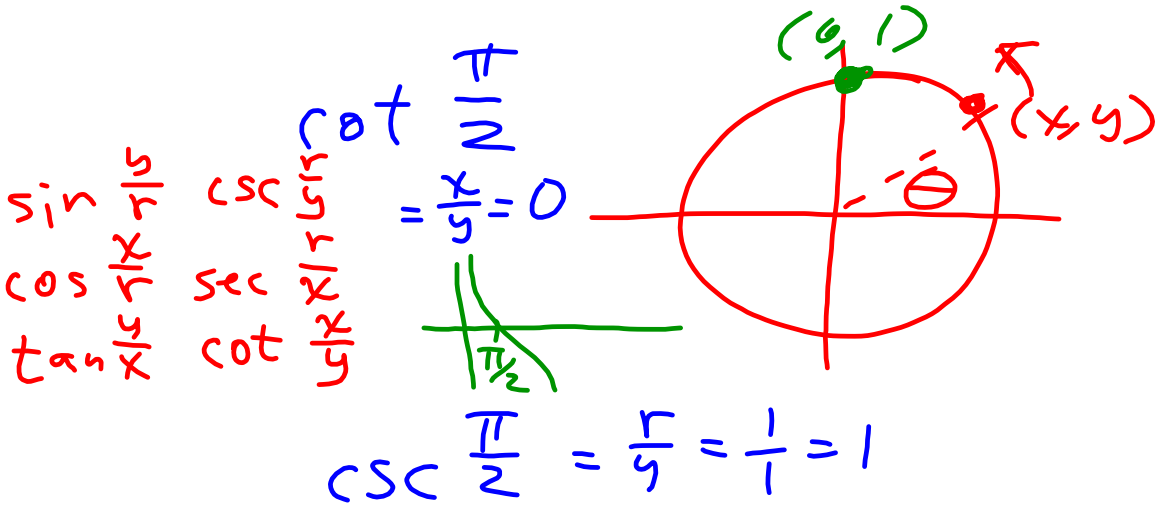
$\frac{dy}{dx} = 0 - \csc^2 x + 2\csc x \cot x$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1$ = slope of tangent

tangent: $y = y_1 + m(x - x_1)$

$y = 2 - (x - \pi/2)$

normal: $y = 2 + (x - \pi/2)$




$\cot \frac{\pi}{2} = \frac{x}{y} = 0$

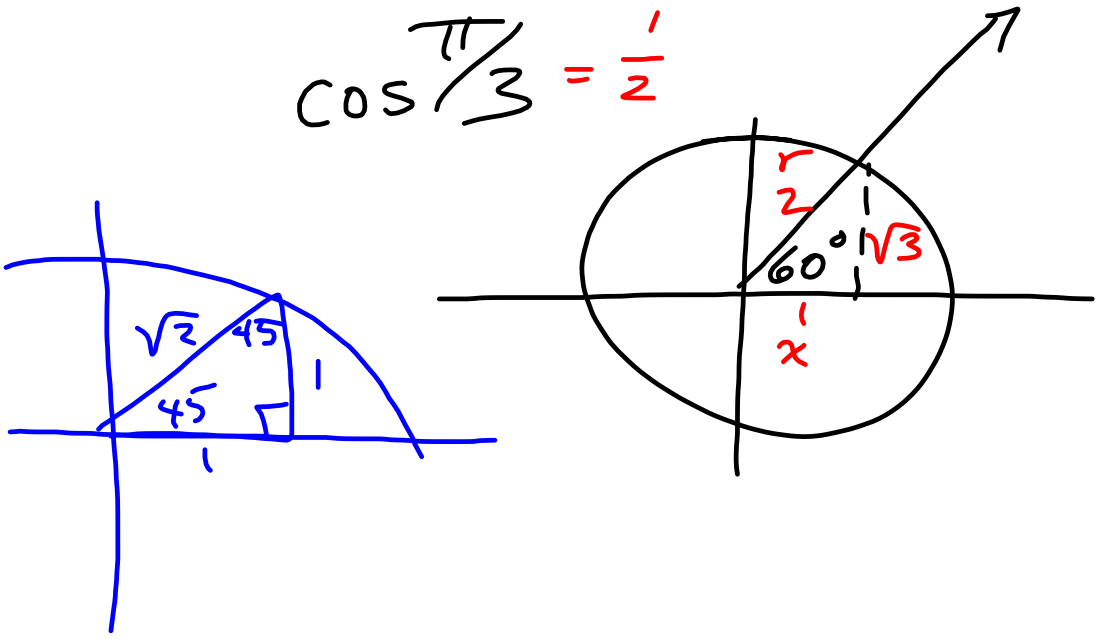
$\sin \frac{y}{r}$ $\csc \frac{r}{y}$

$\cos \frac{x}{r}$ $\sec \frac{r}{x}$

$\tan \frac{y}{x}$ $\cot \frac{x}{y}$

$\csc \frac{\pi}{2} = \frac{r}{y} = \frac{1}{1} = 1$

$\cos \pi/3 = \frac{1}{2}$







Topic: implicit Differentiation builds on chain Rule. For use when you don't see $y = f(x)$

Ex. $y^2 = x$ find $\frac{dy}{dx}$

differentiate both sides with respect to x

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x)$$

$$2y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

Ex. find tangent to circle

$x^2+y^2=25$ @ $(-3,4)$

$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(25)$

$2x+2y\cdot\frac{dy}{dx}=0$

$\frac{dy}{dx}=-\frac{x}{y}\Big|_{(-3,4)}$

$=-\frac{-3}{4}=\frac{3}{4}$

tangent: $y=4+\frac{3}{4}(x+3)$

$y=y_1+m(x-x_1)$

$$\frac{d}{dx}(y^6)=6y^5\cdot\frac{dy}{dx}$$

Ex. $2y = x^2 + \sin(y)$ find $\frac{dy}{dx}$

$$2\frac{dy}{dx} = 2x + \cos y \cdot \frac{dy}{dx}$$

$$2\frac{dy}{dx} - \cos y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(2 - \cos y) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

Ex: tangent to ellipse @ $(-1, 2)$

$$x^2 - xy + y^2 = 7$$

Ex. $2x^3 - 3y^2 = 8$ find $\frac{d^2y}{dx^2}$

$6x^2 - 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x^2}{y}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{x^2}{y}\right)$

Quotient

$u = x^2 \quad v = y$

$u' = 2x \quad v' = \frac{dy}{dx}$

$= \frac{2xy - x^2\frac{dy}{dx}}{y^2}$

$= \frac{2xy - x^2 \cdot \frac{x^2}{y}}{y^2}$

$\vdots$