Ch. 1 pp 22-29
exponential fcns.

p. 170 #9

$s(t) = \sin^{-1}\left(\frac{t}{4}\right)$

want: velocity @ $t=3$

$$v(t) = \frac{1}{\sqrt{1-\frac{t^2}{16}}} \cdot \frac{1}{4}\Bigg|_{t=3}$$

$$= \frac{1}{\sqrt{\frac{7}{16}}} \cdot \frac{1}{4}$$

$$= \frac{4}{\sqrt{7}} \cdot \frac{1}{4} = \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

Quick Review:

1. $\log_5 8$ in terms of ln

change of base: $\frac{\log_a 8}{\log_a 5}$

So: $\frac{\ln 8}{\ln 5}$

to graph $\log_3 x$?

graph $\frac{\ln(x)}{\ln(3)}$

2. 7^x as power of e?

$7 = a^{\log_a(7)} = \log_a(a^7)$

1. $\ln 7$

2.

$7 = e$

$\Rightarrow 7^x = (e^{\ln 7})^x = e^{x \ln 7}$

3. $\ln(e^{\tan x}) = \tan x$

in general: f invertible*

$f^{-1}(f(x)) = f(f^{-1}(x)) = x$

* f has an inverse which is also a fcn.

#4 $\ln(x^2-4) - \ln(x+2)$

$\log a - \log b = \log \frac{a}{b}$

$\log \frac{x^2-4}{x+2} = \log(x-2)$

#5 $\log_2(8^{x-5})$

$8 = 2^3$

$\log_2 8 = 3$

$= \log_2((2^3)^{x-5})$

$= \log_2 2^{3x-15}$

$= 3x-15$

FACT

$$2^3 = 8 \iff \log_2 8 = 3$$

equivalence

2 to power 3 equals 8

power of 2 that gives 8 equals 3

#6 $\dfrac{\log_4 x^{15}}{\log_4 x^{12}}$

$\log a^b = b \log a$

$$\frac{15 \log_4 x}{12 \log_4 x} = \frac{5}{4}$$

#7 $3\ln x - \ln 3x + \ln(12x^2)$

$\ln(x^3) - \ln(3x) + \ln(12x^2)$

$\ln \frac{x^3}{3x}$ $\ln \frac{x^2}{3} + \ln(12x^2)$

$\ln\left(\frac{x^2}{3} \cdot \frac{12x^2}{1}\right)$

$\ln(4x^4)$

#8 $3^x = 19$

(hint: to solve exp. equation, use log, usually ln)

(a) $\log_3 19 = x$

$= \frac{\ln 19}{\ln 3}$ or $= \frac{\log 19}{\log 3}$

(b) $= 2.68$

Examples

$$\frac{d}{dx}\left(e^{3x^2}\right) = e^{3x^2} \cdot 6x$$

$$\frac{d}{dx}\left(3^{\sin x}\right) = 3^{\sin x} \cdot \ln 3 \cdot \cos x$$

Rule: $\frac{d}{dx}(a^x) = a^x \cdot \ln a$

Examples

$$\frac{d}{dx}\left(\ln(\cos x)\right) = \frac{1}{\cos x} \cdot -\sin x$$

$$= -\tan x$$

$$\frac{d}{dx}\left(\log_5(2x)\right) = \frac{1}{2x} \cdot \frac{1}{\ln 5} \cdot 2$$

$$= \frac{1}{x \ln 5}$$