#40 $y^2 = 1 - \frac{2}{x}$

$2y\frac{dy}{dx} = -\frac{d}{dx}(2x^{-1})$

$= -2\frac{d}{dx}(x^{-1})$

$= -2 \cdot -1x^{-2}$

$2y\frac{dy}{dx} = \frac{2}{x^2}$

$\frac{dy}{dx} = \frac{1}{x^2 y} = x^{-2}y^{-1}$

$u = x^{-2} \quad v = y^{-1}$

$u' = -2x^{-3} \quad v' = -y^{-2}\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = -2x^{-3-1}y^{-2-2}\frac{dy}{dx}$

$= -2x^{-3-1}y^{-2-2}x^{-2-1}y$

$= -2x^{-3-1}y^{-4-3}$

$= -\frac{2}{x^3 y} - \frac{1}{x^4 y^3}$

$= -\frac{2xy^2 - 1}{x^4 y^3}$

#41 $y^3 + y = 2\cos x$

$3y^2\frac{dy}{dx} + \frac{dy}{dx} = -2\sin x$

$\frac{dy}{dx}(3y^2 + 1) = -2\sin x$

$\frac{dy}{dx} = \frac{-2\sin x}{3y^2 + 1}$

$u = -2\sin x$

$u' = -2\cos x$

$v = 3y^2 + 1$

$v' = 6y \cdot \frac{-2\sin x}{3y^2 + 1}$

$\frac{d^2y}{dx^2} = \frac{-2\cos x(3y^2+1) - \frac{24y\sin^2 x}{3y^2+1}}{(3y^2+1)^2}$

$= \frac{-2\cos(3y^2+1)(3y^2+1) - 24y\sin^2 x}{(3y^2+1)^3}$

Examples: Find $\frac{dy}{dx}$

① $y = \sin^{-1}(e^x)$

$$y' = \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x$$

$$y' = \frac{e^x}{\sqrt{1-(e^x)^2}}$$

② $y = \ln(\sin x)$

$$y' = \frac{1}{(\sin x)} \cdot \cos x$$

$$= \frac{\cos x}{\sin x} = \cot x$$

(3) $y = e^{\tan^{-1}x}$

$$y' = \frac{e^{\tan^{-1}x}}{1+x^2}$$

(4) $y = \ln(\sin^2 x)$