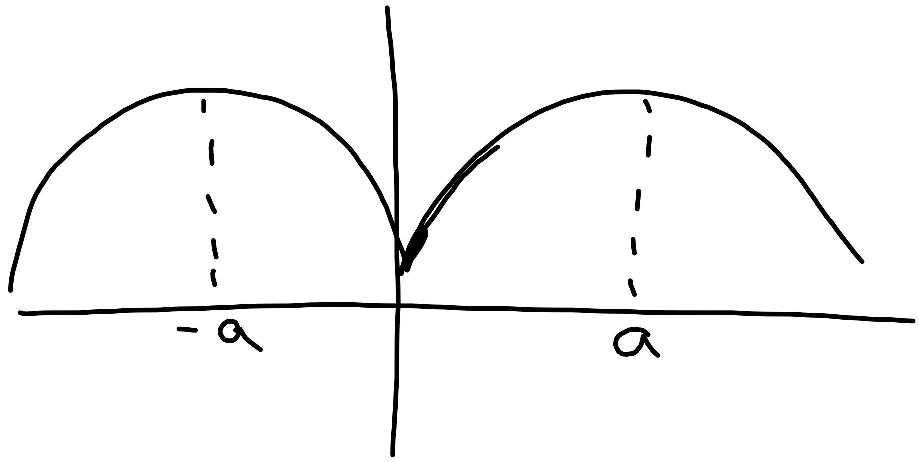


Topic: Mean Value Theorem (MVT) for derivatives.

if $f(x)$ (1) continuous on closed interval $[a, b]$

and (2) differentiable on open interval (a, b)

then: there is an $x = c$ on (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

there is a place where instantaneous r.o.c. = average r.o.c.

MVT: slope of tangent line @ $x=c$ same as slope of secant line.

secant line slope is average r.o.c. $= \frac{4-0}{2-0} = 2$

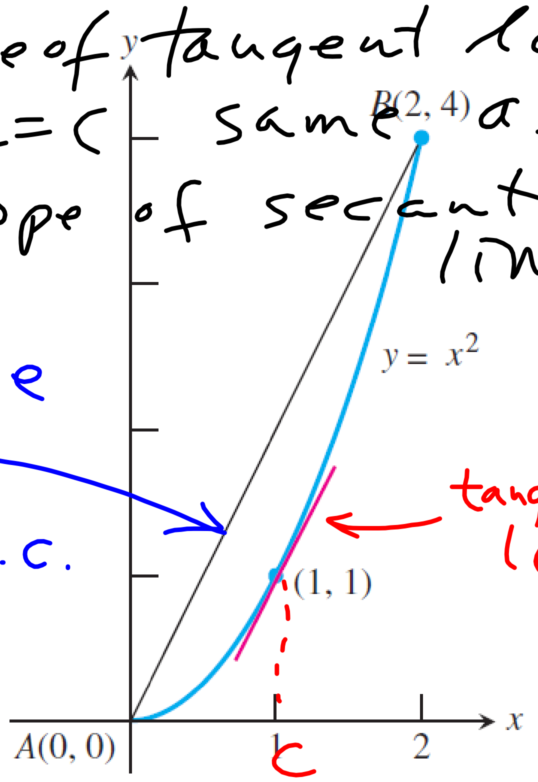


Figure 4.12 (Example 1)

finding c algebraically.

Ex. $y = x^2$ $[0, 2]$

MVT: $y'\big|_{x=c} = \dfrac{y\big|_{x=2} - y\big|_{x=0}}{2-0}$

$$2c = \frac{4-0}{2-0}$$

$$2c = 2$$

$$c = 1$$

Gotchas:

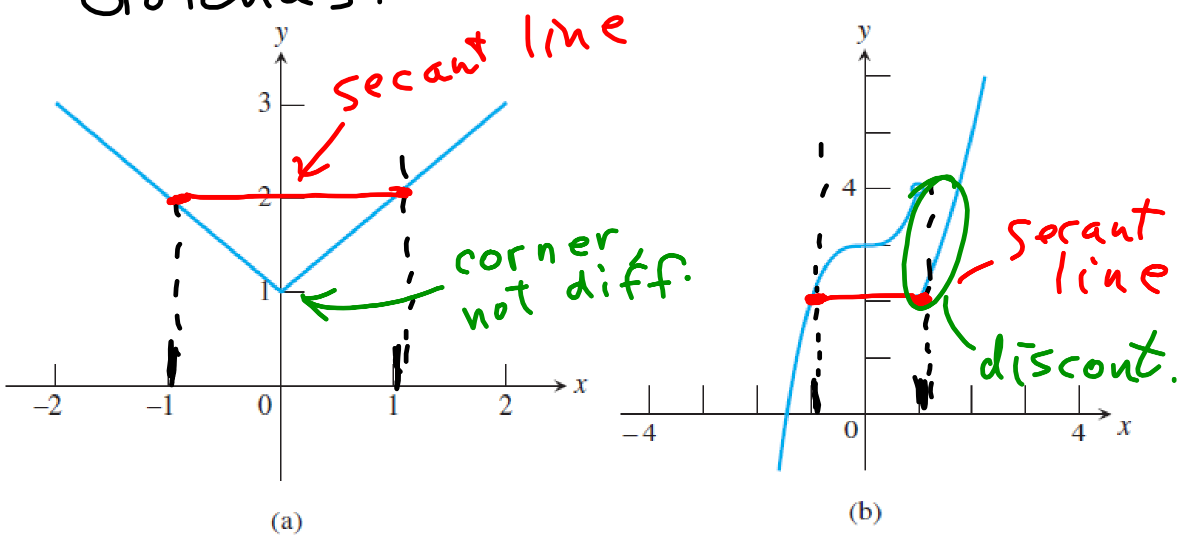


Figure 4.13 For both functions in Example 2, $\dfrac{f(1) - f(-1)}{1 - (-1)} = 0$ but neither function satisfies the conditions of the Mean Value Theorem on the interval $[-1, 1]$. For the function in Example 2(a), there is no number c such that $f'(c) = 0$. It happens that $f'(0) = 0$ in Example 2(b).

352 ft in 8 s.

$\Rightarrow$ must have been going 30 mph for some value of t.

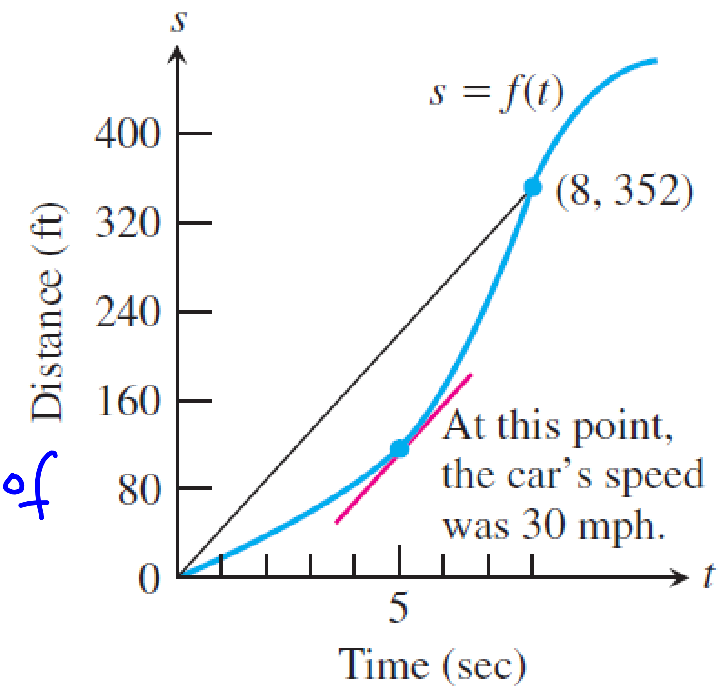


Figure 4.15 (Example 4)

Topic: increasing/decreasing fcns.

- functions are increasing or decreasing on an interval

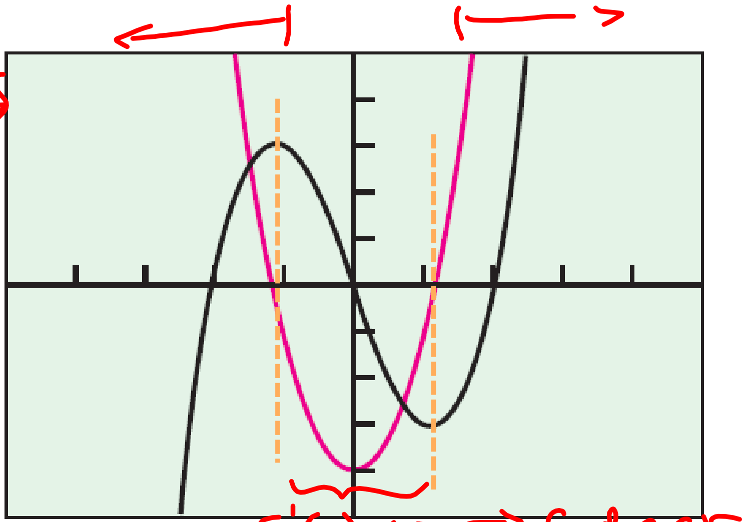
  increasing: $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$

  decreasing: $x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$

- derivative $f'(x)$ is a "marker" for increasing/decreasing

  if $f'(x) > 0$ on an interval, f is increasing.

$f'(x) > 0$
$\Rightarrow f$ increasing



[–5, 5] by [–5, 5]

Figure 4.17 By comparing the graphs of $f(x) = x^3 - 4x$ and $f'(x) = 3x^2 - 4$ we can relate the increasing and decreasing behavior of f to the sign of f'. (Example 6)

#1 – #8 :

1 decide whether you can use MVT

contin. on $[a, b]$

diff. on (a, b)

2 if so, find c

such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

#9 - #10 $\quad y - y_1 = m(x - x_1)$

slope: $\dfrac{y_2 - y_1}{x_2 - x_1}$ or $f'(c)$