increasing: $y' > 0$

decreasing: $y' < 0$

(can only change @ critical points)

concave up: $y'' > 0$

concave down: $y'' < 0$

(can only change when $y''=0$ or y'' undefined)

extreme values: use 1st or 2nd deriv test

inflection points: concavity changes and tangent line

p.256 #3 $y = x^2 e^{x^{-2}}$

$$y' = 2x e^{x^{-2}} + x^2 \cdot (-2x^{-3} e^{x^{-2}})$$

$u = x^2$

$$= 2x e^{x^{-2}} - 2x^{-1} e^{x^{-2}}$$

$u' = 2x$

$$y' = 2e^{x^{-2}}(x - x^{-1})$$

$v = e^{x^{-2}}$

$v' = e^{x^{-2}} \cdot -2x^{-3}$

$= -2x^{-3} e^{x^{-2}}$

y': (−) | (+) | (−) | (+) on the x line, divided at -1, 0, 1

$u = e^{x^{-2}}$

$u' = -2x^{-3} e^{x^{-2}}$

$v = x - x^{-1}$

$v' = 1 + x^{-2}$

$$y'' = 2\left(-2x^{-3} e^{x^{-2}} \cdot (x - x^{-1}) + e^{x^{-2}} \cdot (1 + x^{-2})\right)$$

$$x - \frac{1}{x} = 0$$

$$x = \frac{1}{x}$$

$$x^2 = 1$$

$$x = \pm 1$$

$$-2 - \frac{1}{-2}$$

$$-2 + \frac{1}{2} < 0$$

$$-\frac{1}{2} - (-2)$$

$$-\frac{1}{2} + 2 > 0$$

$$y'' = 2\left(-2x^{-2}e^{x^{-2}} + 2x^{-4}e^{x^{-2}} + e^{x^{-2}} + x^{-2}e^{x^{-2}}\right)$$

$$= 2x^{-4}e^{x^{-2}}\left(-2x^2 + 2 + x^4 + x^2\right)$$

$$y'' = \underbrace{2x^{-4}e^{x^{-2}}}_{\text{never } 0}\underbrace{\left(x^4 - x^2 + 2\right)}_{\text{always } +}$$

undef @ $x = 0$

13. $$y = \begin{cases} e^{-x}, & x \le 0 \\ 4x - x^3, & x > 0 \end{cases}$$

discont. @ $x=0$

$$y' = \begin{cases} -e^{-x}, & x < 0 \\ 4 - 3x^2, & x > 0 \end{cases}$$

y': (−) | 0 | (+) | $\frac{2\sqrt{3}}{3}$ | (−)

$$y'' = \begin{cases} e^{-x}, & x < 0 \\ -6x, & x > 0 \end{cases}$$

y'': (+) | 0 | (−)

$$4 - 3x^2 = 0$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$