MVT: f continuous on $[a, b]$
differentiable on (a, b)

there is $x = c$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

instantaneous rate of change "mean" rate of change

Ex. $f(x) = 3x^2 - 7x + 2$

$[-3, 2]$ $a = -3$, $b = 2$

$f'(x) = 6x - 7$

$f(2) = 3(2)^2 - 7(2) + 2 = 0$

$f(-3) = 3(-3)^2 - 7(-3) + 2 = 50$

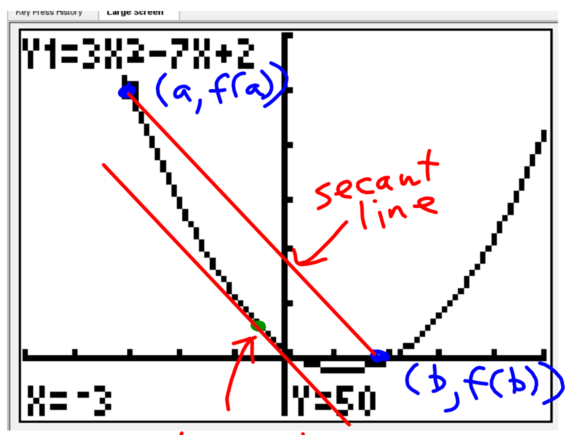
$\frac{f(b) - f(a)}{b - a}$ ← slope of secant line

$= \frac{0 - 50}{2 - (-3)} = \frac{-50}{5} = -10 = 6c - 7$

$6c = -3$

$c = -\frac{1}{2}$

Example



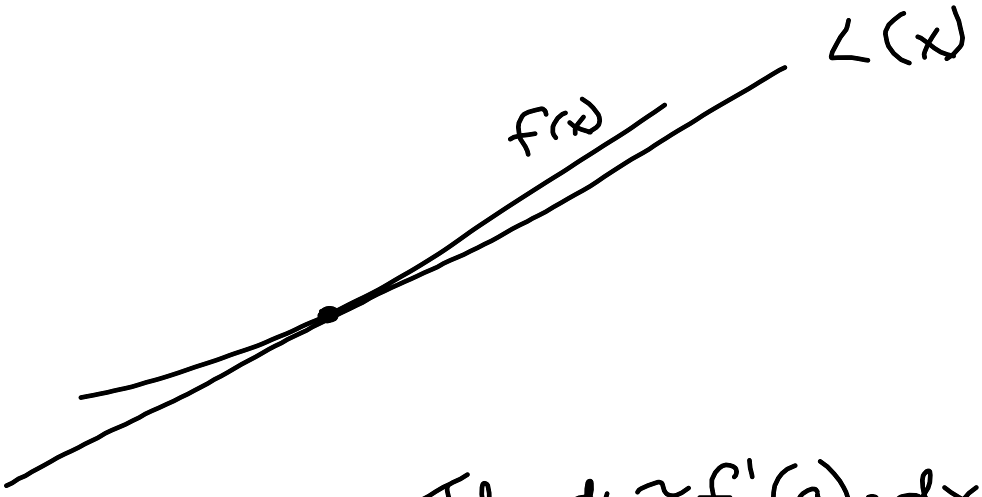
tangent line

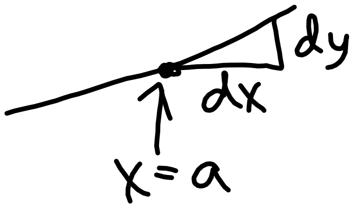
secant line

$y = 0 - 10(x - 2)$

$= -10x + 20$

Linearization





$dy \approx f'(a) \cdot dx$

Extreme values

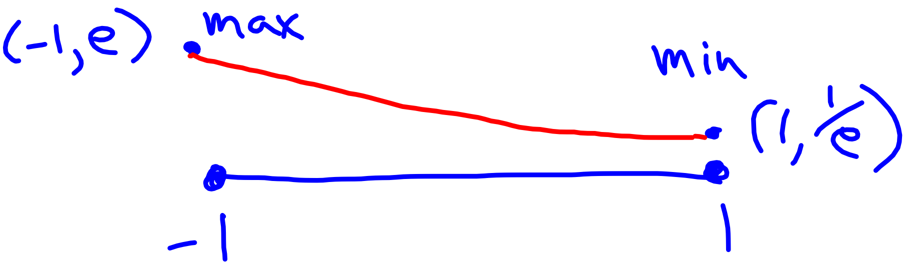
p. 194 # 12

$g(x) = e^{-x} \quad -1 \leq x \leq 1$

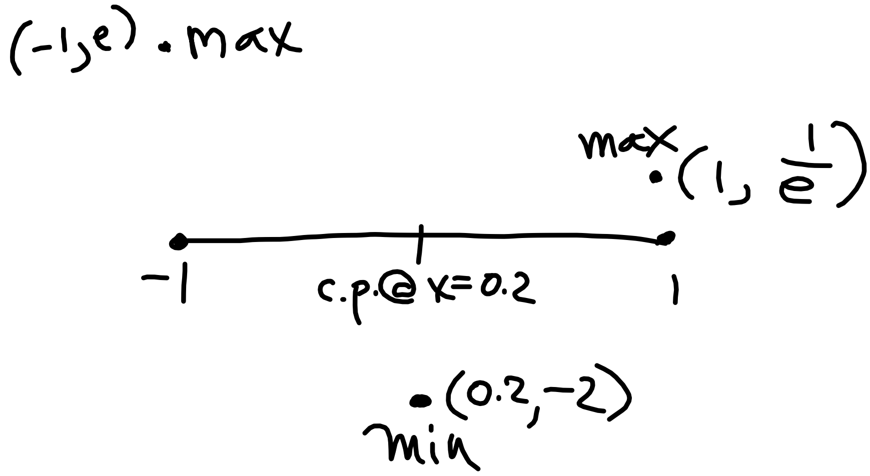
$g'(x) = -e^{-x}$

critical points of g? None

$g(-1) = e^{-(-1)} = e \qquad g(1) = \frac{1}{e}$



What if?



What if: interval is $(-1, 1]$

no global max.

no local max @ $x=-1$

p. 243 #32

1) true change

2) estimated change

3) error $\quad a+dx = 1.1$

$f(x) = x^3 - x \qquad a=1 \qquad dx = 0.1$

1) $f(1.1) - f(1)$

$.23 - 0 = .23$

2) $f'(x) = 3x^2 - 1$

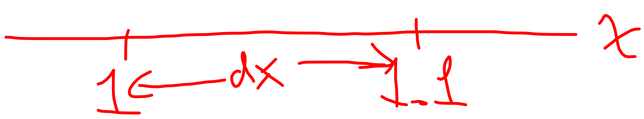
$f'(1) = 2$

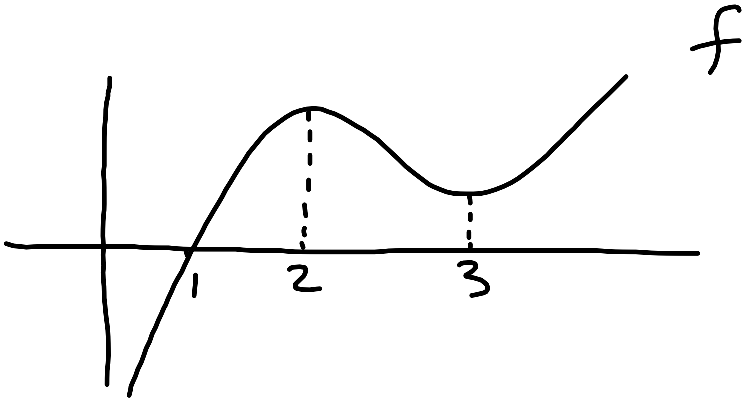
$df \approx f'(1) \cdot dx$

$= 2 \cdot 0.1$

$= 0.20$

3) $|0.23 - 0.20| = .03$



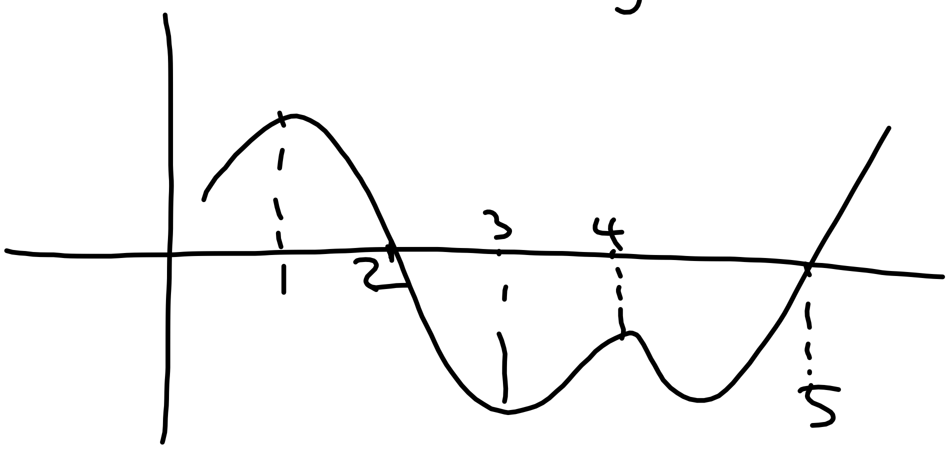


@ $x=2$ $f'(x)=0$

@ $x=1$ $f'(x)>0$

@ $x=2.5$ $f'(x)<0$

$g'(x)$



@ $x=2$, $g(x)$ local max

@ $x=3$ $g(x)$ inflection point