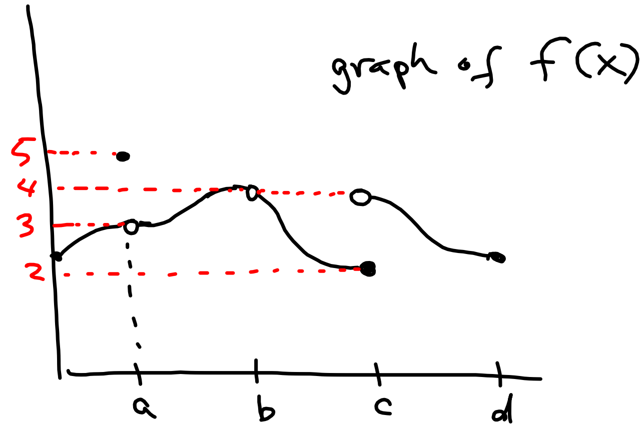
Limits

1. limit of a fcn as x approaches a particular value.
   - from right (right limit)
   - from left (left limit)
   - both (the limit)

$$\lim_{x \to c} f(x) = L$$

x value $\quad$ y value

From a graph



④ $f(a) = 5$ $\quad$ ① $\lim_{x \to a^-} f(x) = 3$

⑤ $f(b) = \text{DNE}^*$

⑥ $\lim_{x \to b} f(x) = 4$ $\quad$ ② $\lim_{x \to a^+} f(x) = 3$

* does not exist $\quad$ ③ $\lim_{x \to a} f(x) =$

When you have $f(x) = \cdots$

- substitution
- factoring.

Ex: $\lim_{x\to 2}(x^2-5x+1) = 2^2-5\cdot 2+1$

$= -5$

(substitution)

Ex: $\lim_{x\to -3}\dfrac{x^2-x-12}{x+3} = \lim_{x\to -3}\dfrac{(x+3)(x-4)}{x+3}$

$= x-4\Big|_{x=-3}$

$= -7$

infinite limits

- vertical asymptotes

  $\lim_{x\to c} f(x) = +\infty, -\infty,$ DNE

  (rational fcns)

- $\lim_{x\to\infty} f(x)$ or $\lim_{x\to -\infty} f(x)$

  horizontal asymptotes

$\frac{p(x)}{q(x)}$ p, q polynomials.

- $\deg(p) > \deg(q)$

$\lim_{x\to\infty} f(x) = +\infty$ or $-\infty$

$\lim_{x\to-\infty} f(x) = +\infty$ or $-\infty$

- $\deg(p) < \deg(q)$: $\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} = 0$
- $\deg(p) = \deg(q)$: $\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} =$ ratio of $\frac{\text{leading coeff } p}{\text{leading coeff } q}$

Ex. $\lim_{x\to\infty} \frac{3x^3 \cdots\cdots}{2x^2 \cdots\cdots} = +\infty$

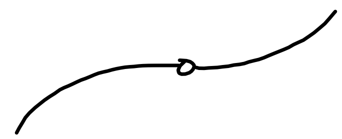
$\lim_{x\to-\infty} \frac{3x^3 \cdots\cdots}{2x^2 \cdots\cdots} = -\infty$

Ex. $\lim_{x\to\pm\infty} \frac{3x^2 \cdots}{2x^3 \cdots} = 0$

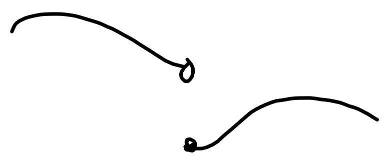
Ex. $\lim_{x\to\pm\infty} \frac{-3x^3 \cdots}{2x^3 \cdots} = -\frac{3}{2}$

Continuity:

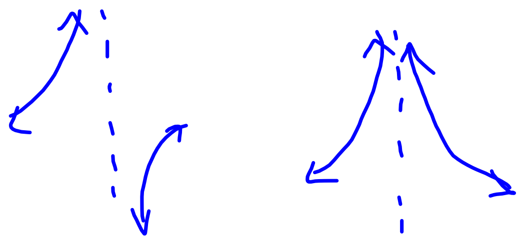
- point discontinuity



- jump discont



- infinite discont.



Rate of change.

average r.o.c. for $f(x)$ on $[a, b]$:

$$\frac{f(b)-f(a)}{b-a}$$

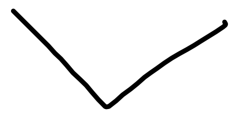
slope of a secant line

DEFINE derivative in these terms:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

→ instantaneous r.o.c. @ $x = a$

For a derivative to exist,
must have continuity.

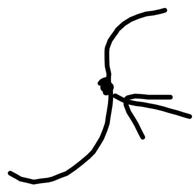
Also: no corner 

(absolute value)

no cusp 

(rational exponent)

no vertical tangent



Derivative: a fcn which
gives instantaneous rate
of change for an interval
(domain)

IVT: Intermediate Value Th.

need: continuity on $[a, b]$

y-value between $f(a)$ and $f(b)$

conclusion: there is an $x = c$ on (a, b) such that

$f(c) =$ y-value

Ex. $f(a) = 2$ $a = 1$

$f(b) = -1$ $b = 5$

The IVT guarantees that one of the following is the value of $f(3)$

(a) -2 (b) 0 (c) 3 (d) -1.5

b/c 0 is between $f(a)$ and $f(b)$

Applications to motion:

position $s = s(t)$

velocity $v = \frac{ds}{dt}$

speed $|v| = \left|\frac{ds}{dt}\right|$

accel. $a = \frac{d^2s}{dt^2} = \frac{dv}{dt}$

sign of s, v, a significant

s: − to + increasing

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$x^2 + xy - y^2 = 1 \quad (2,3)$

tangent, normal line

$2x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$4 + 3 + 2\frac{dy}{dx} - 6\frac{dy}{dx} = 0$

$-4\frac{dy}{dx} = -7$

$\frac{dy}{dx} = \frac{7}{4}$ @(2,3)

Tangent:

$y = 3 + \frac{7}{4}(x-2)$

Normal

$y = 3 - \frac{4}{7}(x-2)$

implicit diff. wrt x

$$\frac{d}{dx}(x \cdot y) = y + x \cdot \frac{dy}{dx}$$

$$\frac{d}{dx}(x^2 \cdot y) = 2xy + x^2 \frac{dy}{dx}$$

$$\frac{d}{dx}(\sin(xy)) = \cos(xy) \cdot \left(y + x \cdot \frac{dy}{dx}\right)$$

off limits

$\sin^{-1}(x)$ etc derivatives.

Parametric (BC)

$$x(t) = 3\cos(4t)$$
$$y(t) = 5\sin(3t)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{15\cos(3t)}{-12\sin(4t)}$$

$$= -\frac{5}{4}\,\frac{\cos(3t)}{\sin(4t)}$$

Chain Rule

$\frac{dy}{dx} = \frac{dy}{df} \cdot \frac{df}{dx}$ $y(f(x))$
(composition)

What if: $f(g(x))$

Find derivative of this fcn when $x = 4$

Know
$g(4) = 3$
$f'(3) = 5$
$g'(4) = -1$

$\boxed{f'(g(x)) \cdot g'(x)}$

$f'(g(4)) \cdot g'(4)$

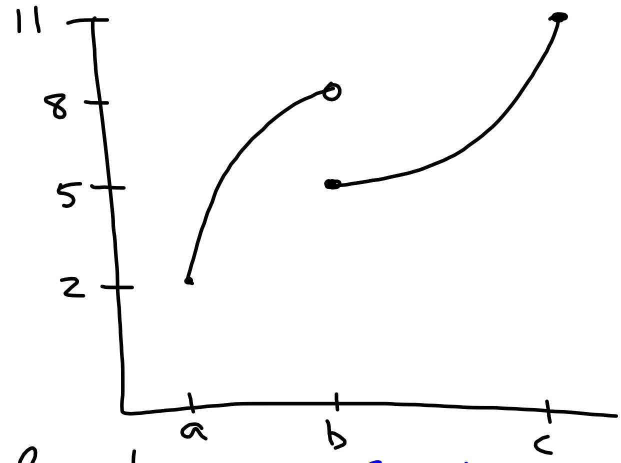
$f'(3) \cdot g'(4)$

$5 \cdot -1$

-5

Be able to write the Chain Rule as:

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$



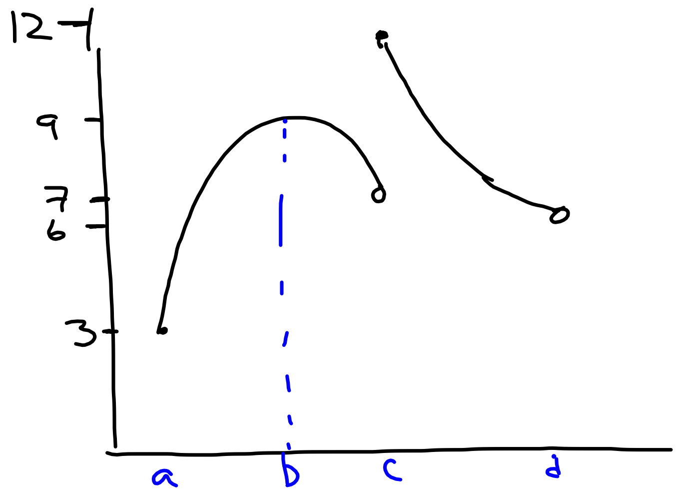
local max : (c, 11)

where what

local min : (a, 2) (b, 5)

global max : (c, 11)

global min : (a, 2)



local max : (c, 12) (b, 9)

local min : (a, 3)

global min : (a, 3)

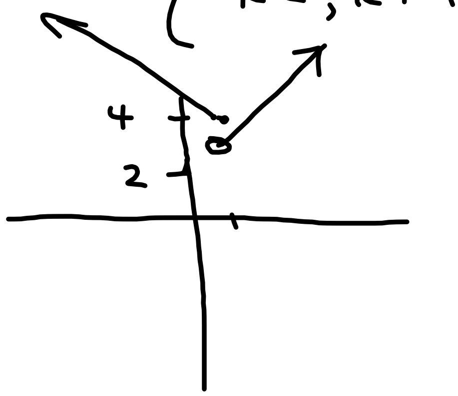
global max : (c, 12)

piece wise
exp / log
differential

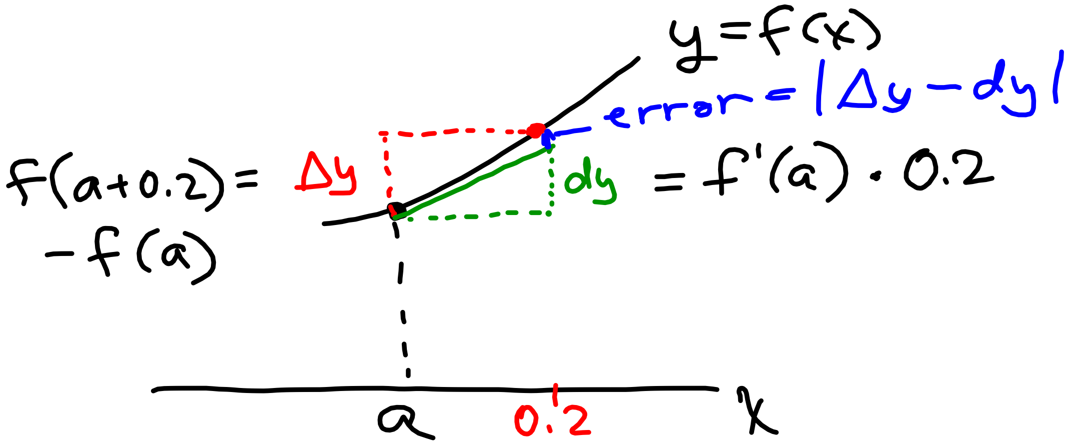
$$\frac{d}{dx}\left(e^{\sin x}\right) = e^{\sin x} \cdot \cos x$$

$$\frac{d}{dx}\left(\ln(3x^2+5)\right) = \frac{1 \cdot 6x}{3x^2+5}$$

Ex. $f(x) = \begin{cases} 5-x, & x \le 1 \\ x+2, & x > 1 \end{cases}$

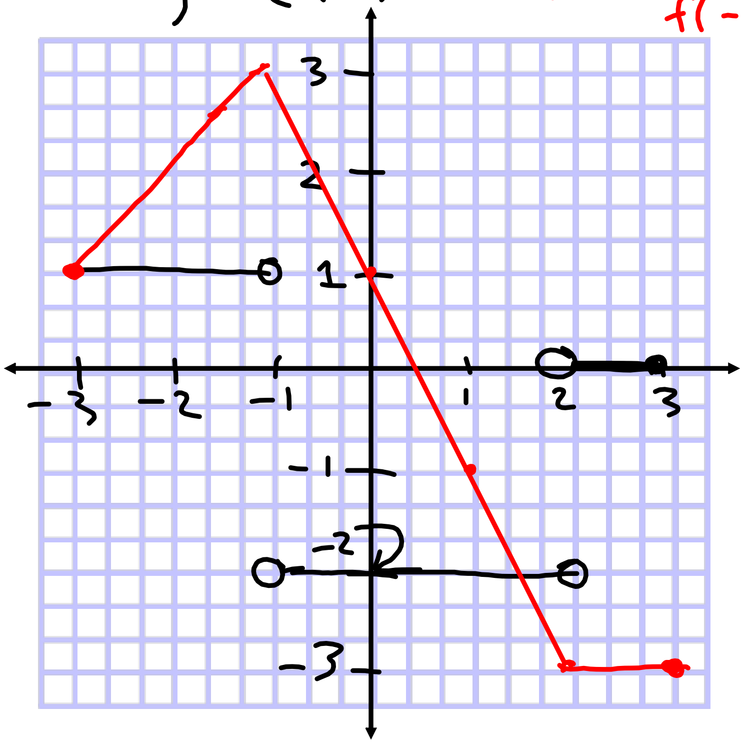


differentials
estimate of Δy

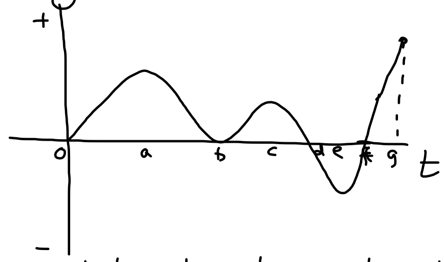


Ex. $f'(x)$

graph f if $f(-3)=1$



object moving $v(t)$



Q: On what intervals is object speeding up? [0, a], [b, c], [d, e], [f, g]

Q: Slowing down?
[a, b], [c, d], [e, f]

Q: At what time(s) is object at rest?
0, b, d, f

Q: At what time(s) does object change directions?
d, f

Related Rates.

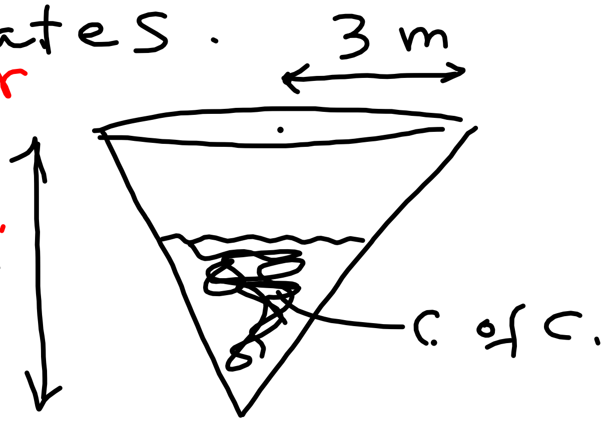
$V = \frac{1}{3}\pi r^2 h \quad h = 2r$

r : radius of c. of c.

h : height of c. of c.

Constant: $\frac{dV}{dt} = \frac{\pi}{2}$.

Want: $\frac{dr}{dt}$ when $h = 1$



Chocolate flowing into cone at a rate of $\frac{\pi}{2}\, m^3/min$. At what rate is the radius of the c. of c. increasing when height of c. of c. = 1 m.

$V = \frac{1}{3}\pi r^2 h \qquad h = 2r$

$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right) \qquad \frac{dh}{dt} = 2\frac{dr}{dt}$

$u = r^2 \quad v = h$

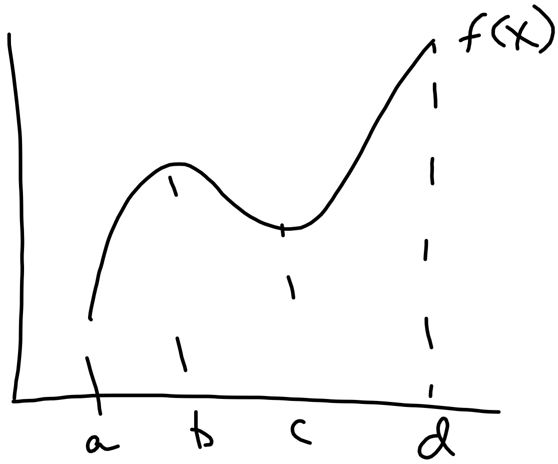
$u' = 2r\frac{dr}{dt} \quad v' = \frac{dh}{dt}$

$\frac{\cancel{\pi}}{2} = \frac{\cancel{\pi}}{3}\left(\cancel{2}\cdot\cancel{\frac{1}{2}}\cdot\frac{dr}{dt}\cdot 1 + \frac{1}{4}\cdot\frac{dh}{dt}\right)$

$\frac{3}{2} = \frac{dr}{dt} + \frac{1}{2}\frac{dr}{dt}$

$\frac{3}{2} = \frac{3}{2}\frac{dr}{dt} \qquad \frac{dr}{dt} = 1\ m/min$

graph of f



where is $f' > 0$ $(a, b), (c, d)$

$f' < 0$ (b, c)

$f' = 0$ b and c