Must Memorize:

BC

$$\frac{dP}{dt} = kP(M - P)$$

same units as P

carrying capacity

$$\Rightarrow P(t) = \frac{M}{1 + Ae^{-Mkt}}$$

based on I.C

Logistic growth

maximum rate of growth occurs when $P = \frac{1}{2} \cdot M$

AB & BC

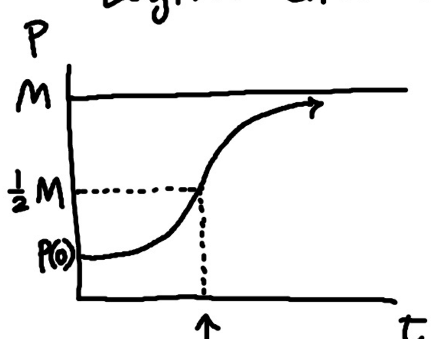
$$\frac{dy}{dt} = ky$$

$$\Rightarrow y(t) = y_0 e^{kt}$$

I.C. @ $t=0$

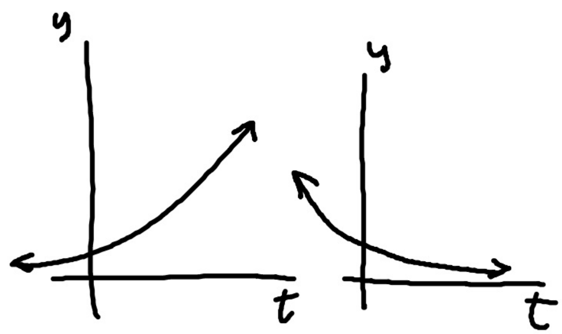
exponential growth, decay

Logistic Growth



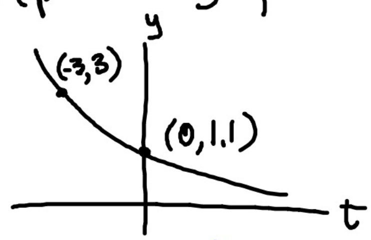
time of maximum growth rate

Exp Growth/decay



Exp. decay p.358

#28



$\frac{dy}{dt} = ky$

$\Rightarrow y(t) = y_0 e^{kt}$

I.C. (0, 1.1) $1.1 = y_0 e^{0\cdot k}$

$1.1 = y_0 \Rightarrow y(t) = 1.1e^{kt}$

I.C. (-3, 3) $3 = 1.1e^{-3\cdot k}$

$\frac{3}{1.1} = e^{-3k}$

$\ln\left(\frac{3}{1.1}\right) = -3k$

$k = \frac{-\ln\left(\frac{3}{1.1}\right)}{3}$

$-\ln\left(\frac{3}{1.1}\right)$

$-(\ln 3 - \ln 1.1)$

$\ln 1.1 - \ln 3$

$k = \frac{\ln 1.1 - \ln 3}{3}$

#7 $\frac{dy}{dx} = \cos x \cdot e^{y+\sin x}$ I.C. (0,0)

separate $\frac{dy}{dx} = \cos x \cdot e^{y} \cdot e^{\sin x}$

Rule on left: $\frac{dy}{e^y} = e^{\sin x} \cdot \cos x \cdot dx$

want

$\int e^u du = e^u + C$ $\int e^{-y} dy = \int \underbrace{e^{\sin x} \cos x \, dx}$

$u = \sin x$
$du = \cos x dx$
$\int e^u du$
$= e^u + C$
$= e^{\sin x} + C$

$u = -y$
$du = -dy$
$dy = -du$
$-\int e^u du$
$-e^u = -e^{-y}$

$-e^{-y} = e^{\sin x} + C$

I.C. $-1 = 1 + C$ $C = -2$

$-e^{-y} = e^{\sin x} - 2$

$e^{-y} = 2 - e^{\sin x}$

$e^{y} = \frac{1}{2 - e^{\sin x}}$

$\boxed{y = \ln\left(\frac{1}{2 - e^{\sin x}}\right)}$

$y = -\ln(2 - e^{\sin x})$