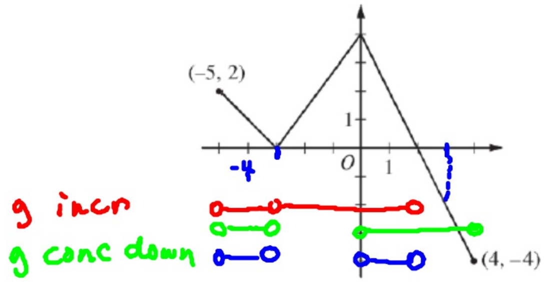
No calculator is allowed for these problems.



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(a) $g(3) = \int_{-3}^{3} f(t)\,dt$

$= \frac{1}{2}(3)(4) + \frac{1}{2}(2)(4) - \frac{1}{2}(1)(2)$

$= 6 + 4 - 1 = \boxed{9}$

$g(x) = \int_{-3}^{x} f(t)\,dt.$

FTC(1)

$g'(x) = f(x)$

$g''(x) = f'(x)$

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

g increasing: $g'(x) > 0$ $(f(x) > 0)$

$(-5,-3)$ & $(-3,2)$

g concave down: $g''(x) < 0$ $f(x)$ decreasing

$f'(x) < 0$

$(-5,-3)$ & $(0,4)$

Both g incr & concave down

$(-5,-3)$ & $(0,2)$

(c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

$h'(3) = \frac{g'(x)5x - 5g(x)}{(5x)^2}$

$u = g(x)$

$V = 5x$

$= \frac{g'(3)15 - 5g(3)}{225}$

$= \frac{-2(15) - 5(9)}{225}$

$= \frac{-30-45}{225}$

$= \boxed{-\frac{1}{3}}$

(d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

$p'(-1)$

$p(x) = f(x^2 - x)$

$p'(x) = f'(x^2 - x) \cdot (2x - 1)$

$p'(-1) = f'(2) \cdot (-3)$

$= (-2)(-3)$

$= 6$

$$L(x) = p(-1) + p'(-1)(x-(-1))$$

for p
@ $x=-1$ $\quad = f((-1)^2-(-1)) + 6(x+1)$

$$= f(2) + 6(x+1)$$

$$y = 6(x+1)$$ ⇐ tangent to p @ $x=-1$

linearization of $f(x)$ @ $x=a$
(tangent line)

$$L(x) = f(a) + f'(a)(x-a)$$